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Physics

For NEET/AIIMS

Module-1

Chapter 1	Basic Mathematics, Unit & Dimension
Chapter 2	Vector
Chapter 3	One-D Motion
Chapter 4	Projectile Motion

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CHAPTER 1

Units and Dimensions, Basic Mathematics

SECTION - A : - UNITS AND DIMENSIONS

PHYSICAL QUANTITIES

All the quantities which are used to describe the laws of physics are known as physical quantities.

Classification : Physical quantities can be classified on the following bases :

I. Based on their directional properties

1. Scalars : The physical quantities which have only magnitude but no direction are called *scalar quantities*.

e.g. mass, density, volume, time, etc.

2. Vectors : The physical quantities which have both magnitude and direction and obey laws of vector alzebra are called *vector quantities*.

e.g. displacement, force, velocity, etc.

II Based on their dependency

1. Fundamental or base quantities : The quantities which do not depend upon other quantities for their complete definition are known as *fundamental or base quantities*.

e.g. length, mass, time, etc.

2. Derived quantities : The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities* .

e.g. Speed (=distance/time), volume, accelaration, force, pressure, etc.

IMPORTANT POINTS

1. Physical quantities can also be classified as dimensional or dimensionless and constant or variable.
2. Some physical quantities can not be completely specified even by specifying their magnitude, unit and direction. These quantities behave neither as a scalar nor as a vector and are called ***tensors***. e.g. Moment of Inertia. It is not a scalar as it has different values in different directions (i.e.about different axes). It is not a vector as changing the sense of rotation (i.e. clockwise or anti clockwise) does not change its value.

Q. Classify the quantities displacement, mass, force, time, speed, velocity, accelaration, moment of intertia, pressure and work under the following categories :

(a) base and scalar (b) base and vector
(c) derived and scalar (d) derived and vector

Ans. (a) mass, time
(b) displacement
(c) speed, pressure, work
(d) force, velocity, accelaration

UNITS OF PHYSICAL QUANTITIES

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

System of Units :

1. FPS or British Engineering system :
In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

2. CGS or Gaussian system :
In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

3. MKS system :
In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

4. International system (SI) of units :
This system is modification over the MKS system and so it is also known as *Rationalised MKS system*. Besides the three base units of MKS system four fundamental and two supplementary units are also included in this system.

SI BASE QUANTITIES AND THEIR UNITS

S. No.	Physical quantity	Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

While defining a base unit or standard for a physical quantity the following characteristics must be considered :
(i) Well defined
(ii) Invariability (constancy)
(iii) Accessibility (easy availability)
(iv) Reproducibility
(v) Convenience in use

Classification of Units : The units of physical quantities can be classified as follows :

1. **Fundamental or base units :**
The units of fundamental quantities are called *base units*. In SI there are seven base units.

2. **Derived units :**
The units of derived quantities or the units that can be expressed in terms of the base units are called derived units.
e.g. unit of speed
$$= \frac{\text{unit of distance}}{\text{unit of time}} = \frac{\text{metre}}{\text{second}} = \text{ms}^{-1}$$
Some derived units are named in honour of great scientists.
e.g. unit of force - newton (N), unit of frequency - hertz (Hz), etc.

3. **Supplementary units :**
In SI two *supplementary units* are also defined viz. radian (rad) for plane angle and steradian (sr) for solid angle.
(i) radian : 1 radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.
(ii) steradian : 1 steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

4. **Practical units :**
Due to the fixed sizes of SI units, some *practical units* are also defined for both fundamental and derived quantities. e.g. light year (ly) is a practical unit of distance (a fundamental quantity) and horse power (hp) is a practical unit of power (a derived quantity).
Practical units may or may not belong to a particular system of units but can be expressed in any system of units.
e.g. 1 mile = 1.6 km = 1.6 × 10^3 m = 1.6 × 10^5 cm.

5. **Improper units :**
These are the units which are not of the same nature as that of the physical quantities for which they are used.
e.g. kg - wt is an improper unit of weight. Here kg is a unit of mass but it is used to measure the weight (force).

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton (N)	poundal
	Work or	erg	joule (J)	ft - poundal
	Energy			
	Power	erg/s	watt (W)	ft - poundal/s

Conversion factors :

To convert a physical quantity from one set of units to the other, the required multiplication factor is called *conversion factor*.
Magnitude of a physical quantity = numeric value (n) × unit (u)
While conversion from one set of units to the other the magnitude of the quantity must remain same. Therefore
$n_1u_1 = n_2u_2$
or nu = constant
or $n \propto \frac{1}{u}$
That is *the numeric value of a physical quantity is inversely proportional to the base unit.*
e.g. 1m = 100 cm = 3.28 ft = 39.4 inch
(SI) (CGS) (FPS)

Some important conversion factors :
Length :
(i) 1 m = 100 cm = 1000 mm = 3.28 ft.
= 39.37 in = 1.0936 yd (yard)
(ii) 1 km = 0.6215 mi (mile)
(iii) 1 mi = 1609 m
(iv) 1 n mi (nautical mile) = 1852 m
(v) 1 in = 2.54 cm
(vi) 1 ft = 12 in = 30.48 cm.
(vii) 1 yd = 3 ft = 91.44 cm.
(viii) 1 μm (micron) = 10^{-6}m

(ix) 1 Å = 10^{-10} m = 0.1 nm

(x) 1 fermi = 10^{-15} m

(xi) 1 bohr radius = 0.529 Å

(xii) 1 AU (Astronomical unit) = 1.49 × 10^{11} m (Average distance between sun and earth)

(xiii) 1 ly (light year) = 9.461 × 10^{15} m (Distance travelled by light in vacuum in one year)

(xiv) 1 parsec or parallactic second = 3.08 × 10^{16}m = 3.26 ly (Distance at which an arc of length 1AU subtends an angle of one second at a point)

Mass :

(i)	1 kg.	= 1000 g = 2.2 lb (pound)
(ii)	1 quintal	= 100 kg
(iii)	1 ton	= 907.2 kg
(iv)	1 metric tonne	= 1000 kg = 10^6 g
(v)	1 lb	= 454 g
(vi)	1 slug	= 14.59 kg
(vii)	1 ounce	= 28.35 g
(viii)	1 amu	= 1.6606 × 10^{-27} kg = 931.5 MeV./c^2
(ix)	1 Chandra Shekhar Limit	= 1.4 M_{sun}

Time :

(i)	1 h	= 60 min	= 3600 s
(ii)	1 d	= 24 h	= 1440 min = 86.4 × 10^3 s
(iii)	1 y	= 365.24 d = 31.56 × 10^6 s	
(iv)	1 shake = 10^{-8} s		

Area :

(i) 1 m^2 = 10^4 cm^2

(ii) 1 km^2 = 0.386 mi^2 = 247 acres

(iii) 1 acres = 43,560 ft^2 = 4047m^2 = 0.4047 hectare

(iv) 1 hectare = 10^4 m^2 = 2.47 acres

(v) 1 barn = 10^{-28} m^2 (for measuring cross-sectional areas in sub-atomic particle collisions)

Volume :

(i) 1 m^3 = 10^6 cm^3 = 10^6 cc = 10^3 L = 35.31 ft^3

(ii) 1 gal (gallon) = 3.786 L (in U.S.A.) or 4.54 L (in U.K.)

Density :

(i) 1 kg m^{-3} = 10^{-3} g cm^{-3} = 10^{-3} kg L^{-1}

Speed :

(i) 1 km h^{-1} = 5/18 or 0.2778 m s^{-1} = 0.6215 mi h^{-1}

(ii) 1 mi h^{-1} = 0.4470 m s^{-1} = 1.609 km h^{-1} = 1.467 ft s^{-1}

(iii) 1 m s^{-1} = 18/5 or 3.6 km h^{-1} = 2.24 mi h^{-1}

Angle and angular speed :

(i) π rad = 180^0

(ii) 1 rad = 180°/π or 57.30^0

(iii) 1^0 = 1.745 × 10^{-2} rad = 60' = 1/360 revolution

(iv) 1 rev = 360° = 2π rad

(v) 1' (min) = 60" (second)

(vi) 1 rev min^{-1} = 0.1047 rad s^{-1} $\approx$ 0.1 rad s^{-1}

(vii) 1 rad s^{-1} = 9.549 rev min^{-1}

Accelaration :

(i) g = 9.8 m s^{-2} (*MKS unit*) = 980 cm s^{-2} (*CGS unit*) = 32 ft s^{-2} (*FPS unit*)

Force :

(i) 1 N = 10^5 dyne = 7.23 poundal

(ii) 1 kg – wt = 1 kg – f = 9.8 N

(iii) 1 g – wt = 1 g – f = 980 dyne

(iv) 1 lb – wt = 1 lb – f = 32 poundal

Pressure :

(i) 1 Pa = 1 N m^{-2} = 10 dyne cm^{-2}

(ii) 1 bar = 10^5 Pa = 10^6 dyne cm^{-2}

(iii) 1 atm = 1.01325 bar = 1.01 × 10^5 Pa = 1.01 × 10^6 dyne cm^{-2} = 760 mm of Mercury

(iv) 1 torr = 1 mm of Hg column = 153.32 Pa

Work energy :

(i) 1 J = 10^7 erg = 0.239 cal

(ii) 1 cal = 4.186 J

(iii) 1 eV = 1.6 × 10^{-19} J

(iv) 1 kWh = 3.6 MJ = 860 kcal

(v) 1 amu = 931 MeV = 1.492 × 10^{-10} J

(vi) 1 Btu (British thermal unit) = 1055 J

Power :

(i) 1 hp (horse power) = 745.7 W $\approx$ 746 W

(ii) 1 W (watt) = 1 J/s

(iii) 1 kW = 1000 W = 1.34 hp

(iv) 1 cal s^{-1} = 4.186 W

Temperature :

(i) K (kelvin) = [°C + 273°] = [°F + 459.67]/1.8 = °R/1.8

(ii) $°F = °C \times \frac{9}{5} + 32$

(iii) °R (rankine) = °F + 459.67

Electric charge :

(i) 1 C (coulomb) = 3 × 10^9 stat coulomb = 0.1 ab coulomb

(ii) 1 esu = 1 stat coulomb = 3.33 × 10^{-10} coulomb

(iii) 1 emu = 1 ab coulomb = 10 coulomb

(iv) 1 A-h = 3600 C (coulomb)

Electric Current :

(i) 1 A (ampere) = 3×10^9 stat ampere (*esu of current*)
= 0.1 ab ampere (*emu of current*)

Radioactivity :

(i) 1 Bq (bacquerel) = 1 dps (disintegration per second)
(ii) 1 Ci (curie) = 3.7×10^{10} dps = 3.7×10^{10} Bq
= 3.7×10^4 Rd
(iii) 1 Rd (rutherford) = 10^6 dps = 10^6 Bq

Others :

(i) 1 weber = 10^8 maxwell (for *Magnetic flux*)
(ii) 1 T (tesla) = 1 weber/m^2 = 10^4 G (gauss)
(for *Magnetic flux density*)
(iii) 1 orested = 79.554 A/m
(for *Intensity of Magnetic field*)
(iv) 1 poiseuille (N-s/m^2 or Pa-s)
= 10 poise (Dyne-s/cm^2) (for *Viscosity*)

EXAMPLE 01 \

The accelaration due to gravity is 9.8 m s^{-2}. Give its value in ft s^{-2}

Sol. As 1m = 3.2 ft
$\therefore$ 9.8 m/s^2 = 9.8×3.28 ft/s^2 = 32.14 ft/s^2 $\approx$ 32 ft/s^2

Q. The value of Gravitational constant G in MKS system is 6.67×10^{-11} N-m^2/kg^2 . What will be its value in CGS system ? **(6.67×10^{-8} cm^3/g s^2)**

Q. Name the smallest and largest units of length.
(fermi and parsec)

Q. Match the type of unit (column A) with its corresponding example (column B)

(A)	(B)
(a) Base unit	(i) N
(b) Derived unit	(ii) hp
(c) Improper unit	(iii) kg - wt
(d) Practical unit	(iv) rad
(e) Supplementary unit	(v) kg

Ans. (a) kg (b) N (c) kg – wt
(d) hp (e) rad.

SECTION - B :- DIMENSIONS

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

1. **Dimensional formula :** The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity. It is written by enclosing the symbols for base quantities with appropriate powers in square brackets i.e. [ ]
e. g. Dim. formula of mass is $[M^1L^0\ T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

2. **Dimensional equation :** The equation obtained by equating a physical quantity with its dimensional formula is called a *dimensional equation*.
e.g. $[v] = [M^0L^1T^{-1}]$
For example $[F] = [MLT^{-2}]$ is a dimensional equation, $[MLT^{-2}]$ is the dimensional formula of the force and the dimensions of force are 1 in mass, 1 in length and –2 in time

3. **Applications of dimensional analysis :**

1. To convert a physical quantity from one system of units to the other :
This is based on a fact that *magnitude of a physical quantity remains same whatever system is used for measurement* i.e.
magnitude = numeric value (n) × unit (u) = constant
or $n_1u_1 = n_2u_2$
So if a quantity is represented by $[M^aL^bT^c]$ then

n_2	=numerical value in II system
n_1	=numerical value in I system
M_1	=unit of mass in I system
M_2	=unit of mass in II system
L_1	=unit of length in I system
L_2	=unit of length in II system
T_1	=unit of time in I system
T_2	=unit of time in II system

$$\boxed{n_2 = n_1\frac{u_1}{u_2} = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c}$$

2. To check the dimensional correctness of a given physical relation :
If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions*.

3. To derive relationship between different physical quantities :
Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Limitations of this method :

(i) This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trignometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like s = ut +at^2 /2 also can't be derived.

(ii) The relation derived from this method gives no information about the dimensionless constants.

Different quantities with units. symbol and dimensional formula.

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	ℓ	Metre or m	M^0LT^0
Area	A	$\ell \times b$	$(\text{Metre})^2$ or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	$(\text{Metre})^3$ or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	-	$F \times t$	N.sec	MLT^{-1}
Work	W	F . d	N . m	ML^2T^{-2}
Energy	KE or U	$K.E. = \frac{1}{2}mv^2$	Joule or J	ML^2T^{-2}
P.E. = mgh				
Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$
Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^0L^0T^0$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	
$M^0L^0T^{-2}$				
Moment of Inertia	I	$I = mr^2$	$kg\text{-}m^2$	ML^2T^0
Frequency	v or f	$f = \frac{1}{T}$	hertz or Hz	$M^0L^0T^{-1}$
Stress	-	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	-	$\frac{\Delta\ell}{\ell};\frac{\Delta A}{A};\frac{\Delta V}{V}$	-	$M^0L^0T^0$
Youngs modulus	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
(Bulk modulus of rigidity)				
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{J}{m^2}$	ML^0T^{-2}
Force constant (spring)	k	F = kx	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S.)	$ML^{-1}T^{-1}$

Gravitation constant	G	$F = \frac{Gm_1 m_2}{r^2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^0L^2T^{-2}$
Temperature	θ	-	Kelvin or K	$M^0L^0T^0\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{Joule}{kg.Kelvin}$	$M^0L^2T^{-2}\theta^{-1}$
Latent heat	L	$Q = mL$	$\frac{Joule}{kg}$	$M^0L^2T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$	$\frac{Joule}{m\,sec\,K}$	$MLT^{-3}\theta^{-1}$
Universal gas constant	R	$PV = nRT$	$\frac{Joule}{mol.K}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	$W = JH$	-	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	-	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$\varepsilon_0 = \frac{1}{4\pi F}.\frac{q_1q_2}{r^2}$	$\frac{(coul.)^2}{N.m^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}T^4A^2$
Electric potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	$E = \frac{F}{q}$	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	$Q = CV$	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	-	$M^0L^0T^0$
Resistance	R	$V = IR$	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	s	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F = qvB\sin\theta$ or $F = BIL$	Tesla or weber/m²	$MT^{-2}A^{-1}$

Magnetic flux	ϕ	$e = \frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L.\frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2\ell$	C.m.	M^0LTA
Magnetic dipole moment	M	M = NIA	amp.m²	$M^0L^2AT^0$

EXAMPLE 02

Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is

$[F] = [M^1 L^1 T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2}$$

$= 1 \times 1000 \times 100 \times \;= 10^5$

$\therefore$ 1 newton = 10^5 dyne.

EXAMPLE 03

Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

Sol. The dimensions of LHS = the dimension of

$T = [M^0 L^0 T^1]$

The dimensions of RHS

$$= \left(\frac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$$

($\because$ 2π is a dimensionless const.)

$$= \left(\frac{L}{LT^{-2}}\right)^{1/2} = (T^2)^{1/2} = [T] = [M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

EXAMPLE 04

Find relationship between speed of sound in a medium (v), the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant & density according to the relation

$v \propto E^a\rho^b$ or $\quad v = KE^a\rho^b$

K = a dimensionless constant of proportionality

Considering dimensions of the quantities

$[v] = M^0 L T^{-1}$

$$[E] = \frac{[stress]}{[strain]} = \frac{\frac{[force]}{[area]}}{\frac{[\Delta\ell]}{[\ell]}} = \frac{\frac{[M^1L^1T^{-2}]}{[L^2]}}{\frac{[L^1]}{[L^1]}} = [M^1L^{-1}T^{-2}]$$

$\therefore \quad [E^a] = [M^a L^{-a} T^{-2a}]$

$\because \quad [\rho] = [mass]/[volume] = [M]/[L^3]$

$= [M^1 L^{-3} T^0]$

$\therefore \quad [\rho^b] = [M^b L^{-3b} T^0]$

Equating the dimensions of the LHS and RHS quantities of equation (1), we get

$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}]\,[M^b L^{-3b} T^0]$

or $[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$

$\therefore \quad a + b = 0,\ -a - 3b = 1$ and $-2a = -1$

On solving $a = \frac{1}{2}$, $b = -\frac{1}{2}$

so the required relation is $v = K\sqrt{\frac{E}{\rho}}$

PRACTICE QUESTION

Q. Match the following :

(a)	Dimensional variable	(p)	π
(b)	Dimensionless variable	(q)	Force
(c)	Dimensional constant	(r)	Angle
(d)	Dimensionless constant	(s)	Gravitational constant

Ans. (a) (p) (b) (q) (c) (r) (d) (s)

Q. Find the dimensions of the following quantities :

(a) Temperature (b) Kinetic energy

(c) Pressure (d) Angular speed

Ans. (a) $[M^0 L^0 T^0 K^1]$ (b) $[M L^2 T^{-2}]$

(c) $[M L^{-1} T^{-2}]$ (d) $[M^0 L^0 T^{-1}]$

Q. Find the dimensions of Planck's constant (h).

Ans. $[M L^2 T^{-1}]$

Q. Centripetal force (F) on a body of mass (m) moving with uniform speed (v) in a circle of radius (r) depends upon m, v and r. Derive a formula for the centripetal force using theory of dimensions.

Ans. $F = K\dfrac{mv^2}{r}$

SECTION - C :- BASIC MATHEMATICS

TRIGONOMETRY

Angle

Consider a revolving line OP. Suppose that it revolves in anticlockwise direction starting from its initial position OX .

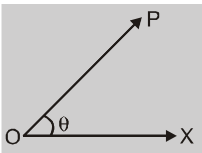


Fig. 1.1

The angle is defined as the amount of revolution that the revolving line makes with its initial position. From fig. the angle covered by the revolving line OP is $\theta = \angle POX$

The angle is **positive :**

If it is traced by the revolving line in anticlockwise direction and is **negative :**

If it is covered in clockwise direction.

$1^\circ = 60'$ (minute)

$1' = 60''$ (second)

1 right angle = 90° (degrees) also

1 right angle = $\dfrac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

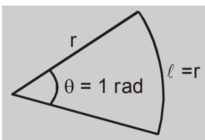


Fig. 1.2

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57^\circ\, 17'\, 45'' \approx 57.3^\circ$$

Trigonometrical ratios (or T ratios)

Let two fixed lines XOX' and YOY' intersecting at right angles to each other at point O. Then,

(i) Point O is called origin.

(ii) XOX' known as X-axis and YOY' are Y-axis.

(iii) Portions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrant respectively.

Consider that the revolving line OP has traced out angle θ (in I quadrant)

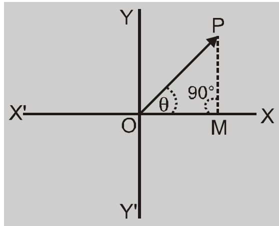


Fig. 1.3

in anticlockwise direction. From P, perpendicular PM on OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called **opposite side or perpendicular** and side OM (making angle θ with hypotenuse) is called **adjacent side or base.**

The three sides of a right angled triangle are connected to each other through six different ratios, called trigonometric ratios or simply T-ratios :

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$$

$$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM}$$

$$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

It can be easily proved that :

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$

Given $\sin\theta = \frac{3}{5}$. Find all the other T-ratios, if θ lies in the first quadrant.

Sol. In Δ OMP, $\sin\theta = \frac{3}{5}$

so MP = 3 and OP = 5

$\because$ OM = $\sqrt{(5)^2-(3)^2}$

= $\sqrt{25-9}$ = $\sqrt{16}$ = 4

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$

$\tan\theta = \frac{MP}{OM} = \frac{3}{4}$

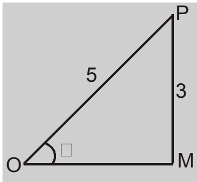


Fig. 1.4

$\cot\theta = \frac{OM}{MP} = \frac{4}{3}$

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$

$\text{cosec}\,\theta = \frac{OP}{MP} = \frac{5}{3}$

Q. If $\sec\theta = \frac{5}{3}$, find all the other T-ratios.

Ans. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{4}$, $\text{cosec}\theta = \frac{5}{4}$

The T-ratios of a few standard angles ranging from 0^0 to 180^0

Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

$\sin(90°+\theta) = \cos\theta$ $\cos(90°+\theta) = -\sin\theta$ $\tan(90°+\theta) = -\cot\theta$	$\sin(180°-\theta) = \sin\theta$ $\cos(180°-\theta) = -\cos\theta$ $\tan(180°-\theta) = -\tan\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$	$\sin(90°-\theta) = \cos\theta$ $\cos(90°-\theta) = \sin\theta$ $\tan(90°-\theta) = \cot\theta$
$\sin(180°+\theta) = -\sin\theta$ $\cos(180°+\theta) = -\cos\theta$ $\tan(180°+\theta) = \tan\theta$	$\sin(270°-\theta) = -\cos\theta$ $\cos(270°-\theta) = -\sin\theta$ $\tan(270°-\theta) = \cot\theta$	$\sin(270°+\theta) = -\cos\theta$ $\cos(270°+\theta) = \sin\theta$ $\tan(270°+\theta) = -\cot\theta$	$\sin(360°-\theta) = -\sin\theta$ $\cos(360°-\theta) = \cos\theta$ $\tan(360°-\theta) = -\tan\theta$

EXAMPLE 06

Find the value of

(i) cos (–60°) (ii) tan 210°

(iii) sin 300° (iv) cos 120°

Sol. (i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180°+30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270°+30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180°-60°) = -\cos 60° = -\frac{1}{2}$

Q. Find the values of :

(i) tan (–30°) (ii) sin 120° (iii) sin 135°

(iv) cos 150° (v) sin 270° (vi) cos 270°

Ans. (i) $-\frac{1}{\sqrt{3}}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{1}{\sqrt{2}}$

(iv) $-\frac{\sqrt{3}}{2}$ (v) -1 (vi) 0

A few important trigonometric formulae

sin (A + B) = sin A cos B + cos A sin B

sin (A – B) = sin A cos B – cos A sin B

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

sin 2 A = 2 sin A cos A

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

cos (A + B) = cos A cos B – sin A sin B

cos (A – B) = cos A cos B + sin A sin B

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

cos 2 A = $\cos^2 A - \sin^2 A$

cos 2 A = $2\cos^2 A - 1 = 1 - 2\sin^2 A$

DIFFERENTIATION

Function

Constant: A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, fractions like π, e, etc are all constants.

Variable: A quantity, which can take different values, is called a variable quantity. A variable is usually represented as x, y, z, etc.

Function: A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as y = f (x)

For example, consider that y is a function of the variable x which is given by

$$y = 3x^2 + 7x + 2$$

If x = 1, then $y = 3(1)^2 + 7(1) + 2 = 12$

and when x = 2, $y = 3(2)^2 + 7(2) + 2 = 28$

Therefore, when the value of variable x is changed, the value of the function y also changes but corresponding to each value of x, we get a single definite value of y. Hence, $y = 3x^2 + 7x + 2$ represents a function of x.

Physical meaning of $\frac{dy}{dx}$

1. The ratio of small change in the function y and the variable x is called the average rate of change of y w.r.t.x.

For example, if a body covers a small distance Δs in small time Δt, then

average velocity of the body, $v_{av} = \frac{\Delta s}{\Delta t}$

Also, if the velocity of a body changes by a small amount Δv in small time Δt, then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$

2. When $\Delta x \to 0$ The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

It is called the instantaneous rate of change of y w.r.t. x.

The *differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.*

Like wise, instantaneous velocity of the body,

$$(v) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

and instantaneous acceleration of the body

$$(a) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Theorems of differentiation :

1. If c = constant, $\frac{d}{dx}(c) = 0$

2. y = c u, where c is a constant and u is a function of x,

$$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$

3. $y = u \pm v \pm w$, where, u, v and w are functions of x,

$$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. y = u v where u and v are functions of x,

$$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

5. $y = \frac{u}{v}$, where u and v are functions of x,

$$\frac{dy}{dx} = \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

6. $y = x^n$, n real number,

$$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

EXAMPLE 07

Find $\frac{dy}{dx}$, when

(i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1}$$

$$= \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

(ii) $y = x^5 + x^4 + 7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4)$$

$$+ \frac{d}{dx}(7) = 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$$

$$= 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

EXAMPLE 07

Find the derivatives of the following :

(i) $(x^3 - 3x^2 + 4)(4x^5 + x^2 - 1)$

(ii) $\frac{9x^5}{x-3}$

Sol. (i) Let $y = (x^3 - 3x^2 + 4)(4x^5 + x^2 - 1)$

$$\frac{dy}{dx} = (x^3 - 3x^2 + 4)\frac{d}{dx}(4x^5 + x^2 - 1)$$

$$+ (4x^5 + x^2 - 1)\frac{d}{dx}(x^3 - 3x^2 + 4)$$

$$= (x^3 - 3x^2 + 4)\left[\frac{d}{dx}(4x^5) + \frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right]$$

$$+ (4x^5 + x^2 - 1)\left[\frac{d}{dx}(x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(4)\right]$$

$$= (x^3 - 3x^2 + 4)\left[4\frac{d}{dx}(x^5) + \frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right] + (4x^5$$

$$+ x^2 - 1)\left[\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) + \frac{d}{dx}(4)\right]$$

$$= (x^3 - 3x^2 + 4)[4 \times 5x^4 + 2x - 0]$$

$$+ (4x^5 + x^2 - 1)[3x^2 - 3 \times 2x + 0]$$

$$= (x^3 - 3x^2 + 4)(20x^4 + 2x) + (4x^5$$

$$+ x^2 - 1)(3x^2 - 6x)$$

$$= 2x(10x^3 + 1)(x^3 - 3x^2 + 4) + 3x$$

$$(x - 2)(4x^5 + x^2 - 1)$$

(ii) Let $y = \frac{9x^5}{x-3}$

$$\frac{dy}{dx} = \frac{(x-3)\frac{d}{dx}(9x^5) - 9x^5\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$= \frac{(x-3) \times 9\frac{d}{dx}(x^5) - 9x^5\left[\frac{d}{dx}(x) - \frac{d}{dx}(3)\right]}{(x-3)^2}$$

$$= \frac{(x-3) \times 9 \times 5x^4 - 9x^5(1-0)}{(x-3)^2}$$

$$= \frac{45x^5 - 135x^4 - 9x^5}{(x-3)^2} = \frac{36x^5 - 135x^4}{(x-3)^2}$$

$$= \frac{9x^4(4x - 15)}{(x-3)^2}$$

Que. Find $\frac{dy}{dx}$ for the following :

1. $y = x^{7/2}$
2. $y = x^{-3}$
3. $y = x$
4. $y = x^5 + x^3 + 4x^{1/2} + 7$
5. $y = 5x^4 + 6x^{3/2} + 9x$
6. $y = ax^2 + bx + c$
7. $y = 3x^5 - 3x - \frac{1}{x}$
8. Given $s = t^2 + 5t + 3$, find $\frac{ds}{dt}$.
9. If $s = ut + \frac{1}{2}at^2$, where u and a are constants. Obtain the value of $\frac{ds}{dt}$.
10. The area of a blot of ink is growing such that after t seconds, its area is given by $A = (3t^2+7)$ cm^2. Calculate the rate of increase of area at t=5 second.
11. The area of a circle is given by $A = \pi r^2$, where r is the radius. Calculate the rate of increase of area w.r.t. radius.

Que. Obtain the differential coeffcient of the following :

12. $(x - 1)(2x + 5)$
13. $(9x^3 - 8x + 7)(3x^5 + 5)$
14. $\dfrac{1}{2x+1}$
15. $\dfrac{3x+4}{4x+5}$
16. $\dfrac{x^2}{x^3+1}$

Ans.

1. $\dfrac{7}{2}x^{5/2}$
2. $-3x^{-4}$
3. 1
4. $5x^4 + 3x^2 + 2x^{-1/2}$
5. $20x^3 + 9x^{1/2} + 9$
6. $2ax + b$
7. $15x^4 - 3 + \dfrac{1}{x^2}$
8. $2t + 5$
9. $u + at$
10. $30\ cm^2\ s^{-1}$
11. $2\pi r$
12. $4x + 3$
13. $216x^7 - 144x^5 + 105x^4 + 135x^2 - 40$
14. $-\dfrac{2}{(2x+1)^2}$
15. $-\dfrac{1}{(4x+5)^2}$
16. $\dfrac{2x - x^4}{(x^3+1)^2}$

Formulae for differential coefficients of trigonometric, logrithmic and exponential functions

1. $\dfrac{d}{dx}(\sin x) = \cos x$
2. $\dfrac{d}{dx}(\cos x) = -\sin x$
3. $\dfrac{d}{dx}(\tan x) = \sec^2 x$
4. $\dfrac{d}{dx}(\cot x) = -\text{cosec}^2 x$
5. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
6. $\dfrac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$
7. $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$
8. $\dfrac{d}{dx}(e^x) = e^x$

INTEGRATION

In integral calculus, the differential coefficient of a function is given. We are required to find the function.

Integration is basically used for summation . Σ is used for summation of discrete values, while $\int$ sign is used for continous function.

If I is integration of f(x) with respect to x then I = $\int f(x)\,dx$ [we can check $\dfrac{dI}{dx} = f(x)$]

$\int f'(x)\,dx = f(x) + c$

Let us proceed to obtain intergral of x^n w.r.t. x.

$\dfrac{d}{dx}(x^{n+1}) = (n+1)x^n$

Since the process of integration is the reverse process of differentiation,

$\int (n+1)x^n dx = x^{n+1}$ or

$(n+1)\int x^n dx = x^{n+1}$ or $\int x^n dx = \dfrac{x^{n+1}}{n+1}$

The above formula holds for all values of n, except n = –1.

It is because, for n = –1, $\int x^n dx = \int x^{-1} dx$

$= \int \dfrac{1}{x} dx$

$\because \quad \dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

$\therefore \quad \int \dfrac{1}{x} dx = \log_e x$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

Few basic formulae of integration

Following are a few basic formulae of integration :

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, Provided $n \neq -1$

2. $\int \sin x dx = -\cos x + c$

$(\because \frac{d}{dx}(\cos x) = -\sin x)$

3. $\int \cos x dx = \sin x + c$

$(\because \frac{d}{dx}(\sin x) = \cos x)$

4. $\int \frac{1}{x} dx = \log_e x + c$

$(\because \frac{d}{dx}(\log_e x) = \frac{1}{x})$

5. $\int e^x dx = e^x + c$

$(\because \frac{d}{dx}(e^x) = e^x)$

EXAMPLE 08

Integrate w.r.t. x. :

(i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}$

Sol. (i) $\int x^{11/2} dx = \frac{x^{11/2+1}}{\frac{11}{2}+1} + c = \frac{2}{13} x^{13/2} + c$

(ii) $\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6} x^{-6} + c$

(iii) $\int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{(p+q)/q} + c$

EXAMPLE 09

Evaluate $\int (x^2 - \cos x + \frac{1}{x})\, dx$

Sol. $= \int x^2 dx - \int \cos x dx + \int \frac{1}{x} dx = \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \frac{x^3}{3} - \sin x + \log_e x + c$

Que. Evaluate the following integrals :

1. $\int x^{15} dx$ **2.** $\int x^{-3/2} dx$

3. $\int (3x^{-7} + x^{-1}) dx$ **4.** $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

5. $\int \left(x + \frac{1}{x}\right) dx$ **6.** $\int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx$

(a and b are constant)

Ans. **1.** $\frac{x^{16}}{16} + c$ **2.** $-2x^{-1/2} + c$

3. $-\frac{x^{-6}}{2} + \log_e x + c$ **4.** $\frac{x^2}{2} + 2x + \log_e x + c$

5. $\frac{x^2}{2} + \log_e x + c$ **6.** $-\frac{a}{x} + b \log_e x + c$

Definite Integrals

When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

If $\frac{d}{dx}(f(x)) = f'(x)$,

then $\int f'(x)dx$ is called indefinite integral

and $\int_a^b f'(x)dx$ is called difinite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below :

$$\int_a^b f'(x)dx = |f(x)|_a^b = f(b) - f(a)$$

EXAMPLE 10

Evaluate the integral : $\int_1^5 x^2\, dx$

Sol. $\int_1^5 x^2\, dx = \left[\frac{x^3}{3}\right]_1^5 = \frac{1}{3}\left[x^3\right]_1^5 = \frac{1}{3}((5)^3 - (1)^3)$

$= \frac{1}{3}(125 - 1) = \frac{124}{3}$

Que. Evaluate the following integrals

7. $\int_R^\infty \frac{GMm}{x^2} dx$ **8.** $\int_{r_1}^{r_2} -k\frac{q_1q_2}{x^2} dx$

9. $\int_u^v Mv\,dv$ **10.** $\int_0^\infty x^{-1/2} dx$

11. $\int_0^{\pi/2} \sin x\, dx$ **12.** $\int_0^{\pi/2} \cos x\, dx$

13. $\int_{-\pi/2}^{\pi/2} \cos x\, dx$

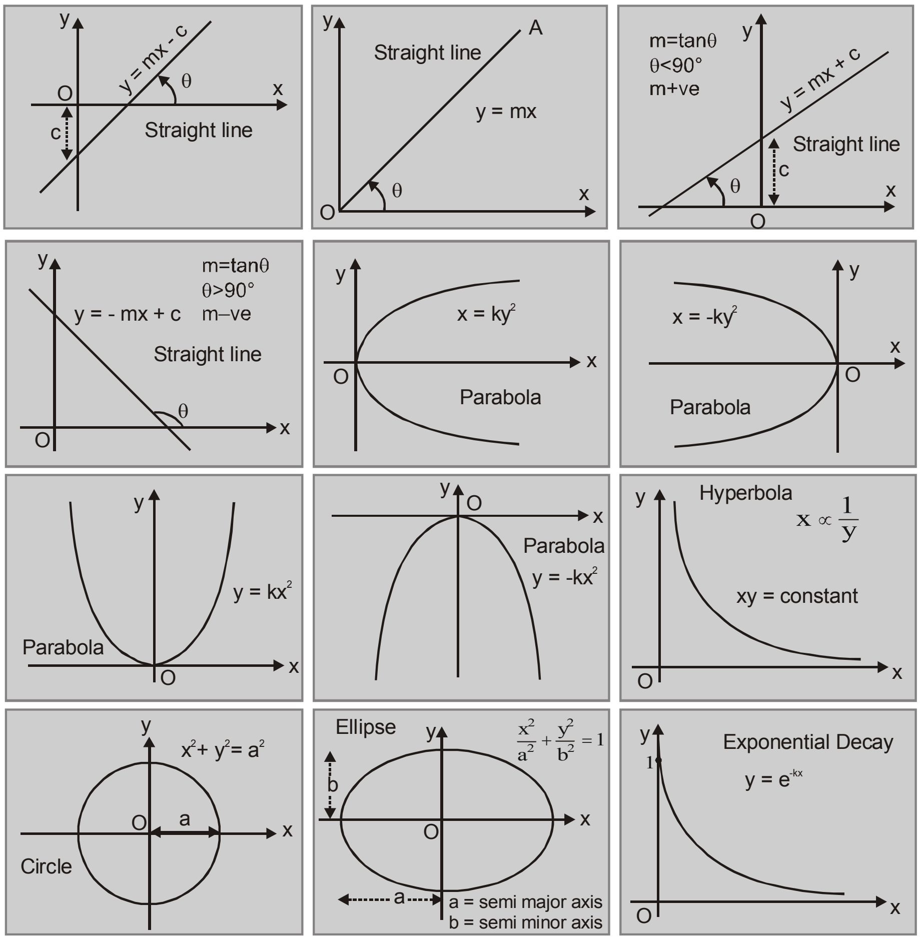
Ans. **7.** $\frac{GMm}{R}$ **8.** $kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

9. $\frac{1}{2}M(v^2 - u^2)$ **10.** ∞

11. 1 **12.** 1

13. 2

SOME STANDARD GRAPHS AND THEIR EQUATIONS



ALGEBRA

Quadratic equation and its solution :

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. Equation $ax^2 + bx + c = 0$ is the general quadratic equation. The general solution of the above quadratic equation or value of variable x is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\text{and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 11

$2x^2 + 5x - 12 = 0$

Sol. By comparision with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$= \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{+6}{4}, \frac{-16}{4}$$

or $x = \frac{3}{2}, -4$

Que. Solve for x :

(i) $10x^2 - 27x + 5 = 0$

(ii) $pqx^2 - (p^2 + q^2)x + pq = 0$

Ans. (i) $\frac{5}{2}, \frac{1}{5}$

(ii) $\frac{p}{q}, \frac{q}{p}$

GEOMETRY

Formulae for determination of area :

1. Area of a square = $(\text{side})^2$
2. Area of rectangle = length × breadth
3. Area of a triangle = $\frac{1}{2}$ base × height

4. Area of trapezoid = $\frac{1}{2}$ (distance between parallel side) × (sum of parallel side)
5. Area enclosed by a circle = πr^2 (r = radius)
6. Surface area of a sphere = $4\pi r^2$ (r = radius)
7. Area of a parallelogram = base × height
8. Area of curved surface of cylinder = $2\pi r\ell$ (r = radius and ℓ = length)
9. Area of ellipse = πab (a and b are semi major and semi minor axes respectively)
10. Surface area of a cube = $6(\text{side})^2$
11. Total surface area of cone = $\pi r^2 + \pi r\ell$

 where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area

Formulae for determination of volume :

1. Volume of a rectangular slab = length × breadth × height = abt

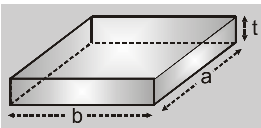


Fig. 1.5

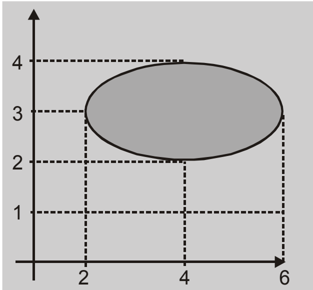
2. Volume of a cube = $(\text{side})^3$
3. Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)
4. Volume of a cylinder = $\pi r^2\ell$ (r = radius and ℓ is length)
5. Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h is height)

Note : $\pi = \frac{22}{7} = 3.14$; $\pi^2 = 9.8776 \approx 10$

and $\frac{1}{\pi} = 0.3182$.

EXAMPLE 12

Calculate the shaded area.



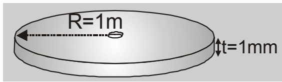
Sol. Shaded area = Area of ellipse = πab

Here $a = 6 - 4 = 2$ and $b = 4 - 3 = 1$

$\Rightarrow$ Area = $\pi \times 2 \times 1 = 2\pi$ units

EXAMPLE 13

Calculate the volume of given disk.



Sol. Volume = $\pi R^2 t = (3.14)(1)^2(10^{-3}) = 3.14 \times 10^{-3}$ m^3

ERROR

SIGNIFICANT FIGURES OR DIGITS

The *significant figures* (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. Rules to find out the number of significant figures :

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule: All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule : The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. e.g. m = 100 kg has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in x = 12.3 = 1.23×10^1 = $.123 \times 10^2$ = 0.0123×10^3 = 123×10^{-1} each term has 3 SF only.

2. Rules for arithmetical operations with significant figures :

I Rule : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $5.0 \times 0.125 = 0.625 = 0.62$

IMPORTANT POINTS

To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in *scientific notation* (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

The change in the unit of measurement of a quantity does not affect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

EXAMPLE 14

Write down the number of significant figures in the following.

(a) 165	3SF (following rule I)
(b) 2.05	3 SF (following rules I & II)
(c) 34.000 m	5 SF (following rules I & V)
(d) 0.005	1 SF (following rules I & IV)
(e) 0.02340 N m^{-1}	4 SF (following rules I, IV & V)
(f) 26900	3 SF (see rule VI)
(g) 26900 kg	5 SF (see rule VI)

EXAMPLE 15

The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. length (ℓ) = 4.234 m

breadth (b) = 1.005 m

thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore area of the sheet

$= 2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2$

$= 2(4.3604739)\ m^2 = 8.720978\ m^2$

Since area can contain a max^m of 3 SF (Rule II of article 4.2) therefore, rounding off, we get

Area = 8.72 m^2

Like wise volume = $\ell \times b \times t$

$= 4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore, rounding off, we get

Volume = 0.0855 m^3

Q. Write the following in scientific notation :

(a) 3256 g (b) .0010 g

(c) 50000 g (5 SF) (d) 0.3204

Q. Give the number of significant figures in the following

(a) 0.165 (b) 4.0026

(c) 0.0256 (d) 165

(e) 0.050 (f) 2.653×10^4

(g) 6.02×10^{23} (h) 0.0006032

Q. Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures.

Q. Subtract 2.5×10^4 from 3.9×10^5 and give the answer to correct number of significant figures.

Q. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures.

Ans.

1. (a) 3.256×10^3 g (b) 1.0×10^{-3} g
   (c) 5.0×10^4 g (d) 3.204×10^{-1}
2. (a) 3 (b) 5
   (c) 3 (d) 3
   (e) 2 (f) 4
   (g) 3 (h) 4
3. 0.882 m^2 (3 SF)
4. 3.6×10^5
5. (a) Total mass = 2.3 kg
   (b) Difference in masses = 0.02g

ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is *rounded off* to