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JEE Class Companion
Physics

For JEE Main and Advanced

Module-1

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Chapter-4 Error

Syllabus

- **Unit Dimension and Basic Mathematics**
  Physics, SI units, Fundamental and derived units. Dimensions of Physical quantities, dimensional analysis and its applications.

- **Kinematics**
  Kinematics in one and two dimensions (Cartesian coordinates only), Projectiles; Relative velocity.

- **Errors**
  Least count, significant figures; Methods of measurement and error analysis for physical quantities pertaining to the experiments.

CHAPTER 1

Unit & Dimensions

SECTION A - UNITS

1. PHYSICAL QUANTITY

The quantites which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

Types of physical quantities :

Fundamental   Derived   Supplementry

1.1 Fundamental

Although the number of physical quantities that we measure is very large, we need only a limited number of units for expressing all the physical quantities since they are interrelated with one another. So, certain physical quantities have been chosen arbitrarily and their units are used for expressing all the physical quantities, such quantities are known as **Fundamental, Absolute** or **Base Quantities** (such as length, time and mass in mechanics)

(i) All other quantites may be expressed in terms of fundamental quantities.

(ii) They are independent of each other and cannot be obtained from one another.

An international body named General Conference on Weights and Measures chose seven physical quantities as fundamental :

(1) length

(2) mass

(3) time

(4) electric current,

(5) thermodynamic temperature

(6) amount of substance

(7) luminous intensity.

Note

These are also called as absolute or base quantities. In mechanics, we treat length, mass and time as the three basic or fundamental quantities.

1.2 Derived :

Physical quantities which can be expressed as combination of base quantities are called as derived quantities.

For example : Speed, velocity, acceleration, force, momentum, pressure, energy etc.

EXAMPLE 1

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{length}}{\text{time}}$$

1.3 Supplementary :

Beside the seven fundamental physical quantities two supplementary quantities are also defined, they are :

(1) Plane angle   (2) Solid angle.

Note

The supplementary quantities have only units but no dimensions.

2. MAGNITUDE :

Magnitude of physical quantity = (numerical value)

Magnitude of a physical quantity is always constant. It is independent of the type of unit.

$$\Rightarrow \text{numerical value} \propto \frac{1}{\text{unit}}$$

or $\quad n_1u_1 = n_2u_2 = \text{constant}$

EXAMPLE 2

Length of a metal rod bar is unchanged whether it is measured as 2 metre or 200 cm.
Observe the change in the Numerical value (from 2 to 200) as unit is changed from metre to cm.

3. UNIT :

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic reference standard called unit.
The units for the fundamental or base quantities are called fundamental or base unit. Other physical quantities are expressed as combination of these base units and hence, called derived units.
A complete set of units, both fundamental and derived is called a system of unit.

3.1. Principle systems of Unit

There are various system in use over the world : CGS, FPS, SI (MKS) etc

Table 1 : Units of some physical quantities in different systems.

	Physical Quantity	System CGS (Gaussian)	MKS (SI)	FPS (British)
Fundamental	Length	centimeter	meter	foot
	Mass	gram	kilogram	pound
	Time	second	second	second
Derived	Force	dyne	newton→N	poundal
	Work or Energy	erg	joule→J	ft-poundal
	Power	erg/s	watt→W	ft-poundal/s

3.2 Supplementary units :

(1) Plane angle : radian (rad)
(2) Solid angle : steradian (sr)

* The SI system is at present widely used throughout the world. In IIT JEE only SI system is followed.

3.3 Definitions of some important SI Units

(i) Metre : 1 m = 1,650, 763.73 wavelengths in vaccum, of radiation corresponding to organ-red light of krypton-86.

(ii) Second : 1 s = 9,192, 631,770 time periods of a particular from Ceasium - 133 atom.

(iii) Kilogram : 1kg = mass of 1 litre volume of water at 4°C

(iv) Ampere : It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of one metre in vacuum produces a force of 2×10^{-7} N/m between them.

(v) Kelvin : 1 K = 1/273.16 part of the thermodynamic temperature of triple point of water.

(vi) Mole : It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12g of carbon - 12.

(vii) Candela : It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right)m^2$ of a black body at the temperature of freezing point under a pressure of 1.013×10^5 N/m^2.

(viii) Radian : It is the plane angle between two radiia of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) Steradian : The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

EXAMPLE 3

Find the SI unit of speed, acceleration

Sol. speed $= \frac{\text{distance}}{\text{time}} = \frac{\text{meter(m)}}{\text{second(s)}}$ = m/s (called as meter per second)

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{\text{displacement / time}}{\text{time}}$$

$$= \frac{\text{displacement}}{(\text{time})^2} = \frac{\text{meter}}{\text{second}^2} = \text{m/s}^2$$

(called as meter per second square)

4. S I PREFIXES

The magnitudes of physical quantities vary order a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

5. GENERAL GUIDELINES FOR USING SYMBOLS FOR SI UNITS, SOME OTHER UNITS, AND SI PREFIXES

(a) Symbols for units of physical quantities are printed/ written in Roman (upright type), and not in italics

For example : 1 N is correct but 1 *N* is incorrect

(b) (i) Unit is never written with capital initial letter even if it is named after a scientist.

For example : SI unit of force is newton (correct) Newton (incorrect)

(ii) For a unit named after a scientist, the symbol is a capital letter.

But for other units, the symbol is NOT a capital letter.

For example :

force	→	newton (N)
energy	→	joule (J)
electric current	→	ampere (A)
temperature	→	kelvin (K)
frequency	→	hertz (Hz)

For example :

length	→	meter (m)
mass	→	kilogram (kg)
luminous intensity	→	candela (cd)
time	→	second (s)

Note

The single exception is L, for the unit litre.

(c) Symbols for units do not contain any final full stop all the end of recommended letter and remain unaltered in the plural, using only singular form of the unit.

For example :

Quantity	Correct	Incorrect
25 centimeters	25 cm	25 c m 25 cms.

(d) Use of solidus (/) is recommended only for indicating a division of one letter unit symbol by another unit symbol. Not more than one solidus is used.

For example :

Correct	Incorrect
m/s^2	m / s / s
N s/m^2	N s / m/ m
J/K mol	J / K / mol
kg/m s	kg / m / s

(e) Prefix symbols are printed in roman (upright) type without spacing between the prefix symbol and the unit symbol. Thus certain approved prefixes written very close to the unit symbol are used to indicate decimal fractions or multiples of a SI unit, when it is inconveniently small or large.

For example

megawatt	1 MW = 10^6 W
centimetre	1 cm = 10^{-2} m
kilometre	1 km = 10^3 m
millivolt	1 mV = 10^{-3} V
kilowatt-hour	1 kW h = 10^3 W h = 3.6 M J = 3.6 × 10^6 J
microampere	$1\mu A = 10^{-6} A$
angstrom	$1 Å = 0.1 nm == 10^{-10} m$
nanosecond	1 ns = 10^{-9} s
picofarad	1 pF = 10^{-12} F
microsecond	$1\mu s = 10^{-6}$ s
gigahertz	1 GHz = 10^9 Hz
micron	1 μm = 10^{-6} m

The unit 'fermi', equal to a femtometre or 10^{-15} m has been used as the convenient length unit in nuclear studies.

(f) When a prefix is placed before the symbol of a unit, the combination of prefix and symbol is considered as a new symbol, for the unit, which can be raised to a positive or negative power without using brackets. These can be combined with other unit symbols to form compound unit.

For example :

Quantity	Correct	Incorrect
cm^3	$(cm)^3 = (0.01\ m)^3 = (10^{-2} m)^3 = 10^{-6}\ m^3$	$0.01\ m^3$ or $10^{-2}\ m^3$
mA^2	$(mA)^2 = (0.001\ A)^2 = (10^{-3} A)^2 = 10^{-6}\ A^2$	$0.001\ A^2$

(g) A prefix is never used alone. It is always attached to a unit symbol and written or fixed before the unit symbol.

For example :

$10^3/m^3$ = $1000/m^3$ or 1000 m^{-3}, but not k/m^3 or k m^{-3}.

(h) Prefix symbol is written very close to the unit symbol without spacing between them, while unit symbols are written separately with spacing with units are multiplied together.

For example :

Quantity	Correct	Incorrect
1 ms^{-1}	1 metre per second	1 milli per second
1 ms	1 millisecond	1 metre second
1 C m	1 coulomb metre	1 centimetre
1 cm	1 centimetre	1 coulomb metre

(i) The use of double profixes is avoided when single prefixes are available.

For example :

Quantity	Correct	Incorrect
10^{-9} m	1 nm (nanometre)	1 m μm (milli micrometre)
10^{-6} m	1 μm (micron)	1 m m m (milli millimetre)
10^{-12} F	1 pF (picofarad)	1 μ μ F (micro microfarad)
10^9 F	1 GW (giga watt)	1 kM W (kilo megawatt)

(j) The use of a combination of unit and the symbols for unit is avoided when the physical quantity is expressed by combining two or more units.

Quantity	Correct	Incorrect
joule per mole Kelvin	J/mol K or J mol^{-1} K^{-1}	Joule / mole K or J/mol Kelvin or J/mole K
newton metre second	N m s	newton m second or N m second or N metre s or newton metre s

5.1. Characteristics of base units or standards :

(A) Well defined (B) Accessibility
(C) Invariability (D) Convenience in use

5.2 Some special types of units :

1. 1 Micron (1μ) = 10^{-4} cm = 10^{-6} m (length)
2. 1 Angstrom (1 Å) = 10^{-8} cm = 10^{-10}m (length)
3. 1 fermi (1f) = 10^{-13} cm = 10^{-15} m (length)
4. 1 inch = 2.54 cm (length)
5. 1 mile = 5280 feet = 1.609 km (length)
6. 1 atmosphere = 10^5 N/m^2 = 76 torr = 76 mm of Hg pressure (pressure)
7. 1 litre = 10^{-3} m^3 = 1000 cm^3 (volume)
8. 1 carat = 0.0002 kg (weight)
9. 1 pound (Ib) = 0.4536 kg (weight)

Note

The student can now attempt section A from exercise.

Section B - Dimensions

6. DIMENSIONS

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.

For example, $\text{density} = \dfrac{\text{mass}}{\text{volume}} = \dfrac{\text{mass}}{(\text{length})^3}$

or $\text{density} = (\text{mass})\,(\text{length})^{-3}$ ...(i)

Thus, the dimensions of density are 1 in mass and –3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by M, length by L, time by T and electric current by A.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively.

The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

$[\sin\theta] = [\cos\theta] = [\tan\theta] = [e^x] = [M^0L^0T^0]$

7. DIMENSIONAL FORMULA

It is an expression which shows how and which of the fundalmental units are required to represent the unit of physical quantity.

Different quantities with units. symbol and dimensional formula.

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	ℓ	Metre or m	M^0LT^0
Area	A	$\ell \times b$	(Metre)^2 or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	(Metre)^3 or m^3	$M^0L^3T^0$
Velocity	v	$v = \dfrac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \dfrac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	-	F × t	N.sec	MLT^{-1}
Work	W	F . d	N . m	ML^2T^{-2}
Energy	KE or U	$K.E. = \dfrac{1}{2}mv^2$	Joule or J	ML^2T^{-2}
P.E. = mgh				
Power	P	$P = \dfrac{W}{t}$	watt or W	ML^2T^{-3}

Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$
Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^0L^0T^0$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^0L^0T^{-2}$
Moment of Inertia	I	$I = mr^2$	$kg\text{-}m^2$	ML^2T^0
Frequency	v or f	$f = \frac{1}{T}$	hertz or Hz	$M^0L^0T^{-1}$
Stress	-	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	-	$\frac{\Delta\ell}{\ell}; \frac{\Delta A}{A}; \frac{\Delta V}{V}$	-	$M^0L^0T^0$
Youngs modulus (Bulk modulus of rigidity)	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m}; \frac{J}{m^2}$	ML^0T^{-2}
Force constant (spring)	k	F = kx	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S.)	$ML^{-1}T^{-1}$
Gravitation constant	G	$F = \frac{Gm_1 m_2}{r^2}$	$\frac{N - m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^0L^2T^{-2}$
Temperature	θ	-	Kelvin or K	$M^0L^0T^0\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{\text{Joule}}{\text{kg. Kelvin}}$	$M^0L^2T^{-2}\theta^{-1}$
Latent heat	L	Q = mL	$\frac{\text{Joule}}{\text{kg}}$	$M^0L^2T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$	$\frac{\text{Joule}}{\text{m sec K}}$	$MLT^{-3}\theta^{-1}$
Universal gas constant	R	PV = nRT	$\frac{\text{Joule}}{\text{mol.K}}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	W = JH	-	$M^0L^0T^0$

Charge	Q or q	$I=\frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	-	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$\varepsilon_0=\frac{1}{4\pi F}.\frac{q_1q_2}{r^2}$	$\frac{(coul.)^2}{N.m^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}T^4A^2$
Electric potential	V	$V=\frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	$E=\frac{F}{q}$	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	$Q=CV$	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}$	-	$M^0L^0T^0$
Resistance	R	$V=IR$	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S=\frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho=\frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	s	$\sigma=\frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F=qvB\sin\theta$ or $F=BIL$	Tesla or weber/m^2	$MT^{-2}A^{-1}$
Magnetic flux	φ	$e=\frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B=\mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B=\frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e=L.\frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p=q\times 2\ell$	C.m.	M^0LTA
Magnetic dipole moment	M	$M=NIA$	amp.m^2	$M^0L^2AT^0$

Note

The student can now attempt section B from exercise.

SECTION C - USE OF DIMENSIONS

8. USE OF DIMENSIONS

Theory of dimensions have following main uses :

8.1 Conversion of units :

This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, i.e.,

$n[u]$ = constant or $n_1[u_1] = n_2[u_2]$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then we can write.

$$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c] \quad ...(i)$$

Here n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

EXAMPLE 4

The value of gravitation constant is $G = 6.67 \times 10^{-11}$ Nm²/kg² in SI units. Convert it into CGS system of units.

Sol. The dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

Using equation number (i), i.e.,

$$n_1[M_1^{-1} L_1^3 T_1^{-2}] = n_2[M_2^{-1} L_2^3 T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

$M_1 = 1$ kg, $M_2 = 1$ g $= 10^{-3}$ kg

$L_1 = 1$ m, $L_2 = 1$cm $= 10^{-2}$ m,

$T_1 = T_2 = 1$s

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\text{kg}}{10^{-3}\text{kg}}\right]^{-1}\left[\frac{1\text{m}}{10^{-2}\text{m}}\right]^{3}\left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

or $n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm²/g².

8.2 To check the dimensional correctness of a given physical equation :

Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

EXAMPLE 5

Show that the expression of the time period T of a simple pendulum of length l given by $T = 2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

Dimensionally $[T] = \sqrt{\frac{[L]}{[LT^{-2}]}} = [T]$

As in the above equation, the dimensions of both sides are same. The given formula is dimensionally correct.

8.3 Principle of Homogeneity of Dimensions.

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Note

The physical quantities separated by the symbols +, –, =, >, < etc., have the same dimensions.

EXAMPLE 6

The velocity v of a particle depends upon the time t according to the equation $v = a + bt + \frac{c}{d+t}$. Write the dimensions of a, b, c and d.

Sol. From principle of homogeneity

$[a] = [v]$

or $[a] = [LT^{-1}]$ **Ans.**

$[bt] = [v]$

or $[b] = \frac{[v]}{[t]} = \frac{[LT^{-1}]}{[T]}$

or $[b] = [LT^{-2}]$

Similarly, $[d] = [t] = [T]$ **Ans.**

Further, $\frac{[c]}{[d+t]} = [v]$

or $[c] = [v]\,[d + t]$

or $[c] = [LT^{-1}]\,[T]$

or $[c] = [L]$ **Ans.**

8.4 To establish the relation among various physical quantities :

If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors. Let us take an example.

EXAMPLE 7

The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length l of the string and the mass per unit length μ of string. Derive the formula for frequency.

Sol. Suppose, that the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c. Then.

$$f \propto [F]^a [l]^b [\mu]^c$$

or $$f = k[F]^a [l]^b [\mu]^c$$

Here, k is a dimensionless constant. Thus,

$$[f] = [F]^a [l]^b [\mu]^c$$

or $[M^0 L^0 T^{-1}] = [MLT^{-2}]^a [L]^b [ML^{-1}]^c$

or $[M^0L^0T^{-1}] = [M^{a+c} L^{a+b-c} T^{-2a}]$

For dimensional balance, the dimension on both sides should be same.

Thus, $a + c = 0$ ...(ii)

$a + b - c = 0$ ...(iii)

and $-2a = -1$ ...(iv)

Solving these three equations, we get

$$a = \frac{1}{2},\quad c = -\frac{1}{2} \quad \text{and } b = -1$$

Substituting these values in Eq. (i), we get

$$f = k(F)^{1/2}(l)^{-1}(\mu)^{-1/2} \quad \text{or} \quad f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$

Experimentally, the value of k is found to be $\frac{1}{2}$

Hence, $$f = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

8.5 Limitations of Dimensional Analysis

The method of dimensions has the following limitations :

(i) By this method the value of dimensionless constant can not be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

Note

The student can now attempt section C from exercise.

SECTION D - BASIC MATHEMATICS

9. MENSURATION FORMULAS :

r : radius ; d = diameter ;

V = Volume S.A = surface area

(a) Circle

Perameter : $2\pi r = \pi d$, Area : $\pi r^2 = \frac{1}{4}\pi d^2$

(b) Sphere

Surface area = $4\pi r^2 = \pi d^2$,

Volume = $\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

(c) Spherical Shell (Hollow sphere)

Surface area = $4\pi r^2 = \pi d^2$

Volume of material used = $(4\pi r^2)(dr)$,

dr = thickness

(d) Cylinder

Lateral area = $2\pi rh$

$V = \pi r^2 h$

Total area = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

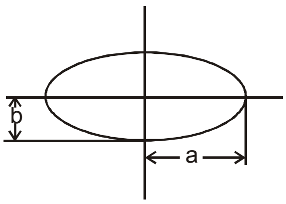
(e) Cone

Lateral area = $\pi r\sqrt{r^2 + h^2}$ h = height

Total area = $\pi r\left(\sqrt{r^2 + h^2} + r\right)$ $V = \frac{1}{3}\pi r^2 h$

(f) Ellipse

Circumference $\approx 2\pi\sqrt{\frac{a^2 + b^2}{2}}$



area = πab

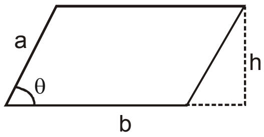
a = semi major axis

b = semi minor axis

(g) Parallelogram

$A = bh = ab\sin\theta$

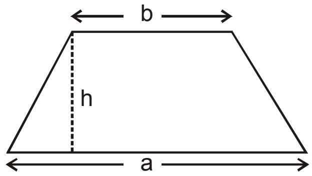
a = side ; h = height ; b = base



θ = angle between sides a and b

(h) Trapezoid

area = $\frac{h}{2}(a + b)$



a and b parallel sides

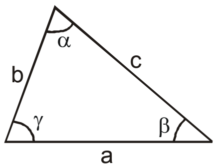
h = height

(i) Triangle

area = $\frac{bh}{2} = \frac{ab}{2}\sin\gamma = \sqrt{s(s-a)(s-b)(s-c)}$

a, b, c sides are opposite to angles α, β, γ

b = base ; h = height

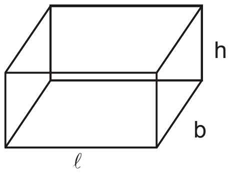


$s = \frac{1}{2}(a + b + c)$

(j) Rectangular container

lateral area = $2(\ell b + bh + h\ell)$

$V = \ell bh$



side ℓ,b,h

Mathematics is the language of physics. It becomes easier to describe, understand and apply the physical principles, if one has a good knowledge of mathematics.

10. LOGARITHMS :

(i) $e \approx 2.7183$

(ii) If $e^x = y$, then $x = \log_e y = \ln y$

(iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 2.303 \log_{10} y$

(v) $\log(ab) = \log(a) + \log(b)$

(vi) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(vii) $\log a^n = n \log(a)$

11. TRIGONOMETRIC PROPERTIES :

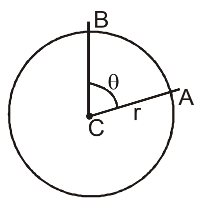
(i) Measurement of angle & relationship between degrees & radian

In navigation and astronomy, angles are measured in degrees, but in calculus it is best to use units called radians because of they simplify later calculations.

Let ACB be a central angle in **circle** of radius r, as in figure.

Then the angle ACB or θ is defined in radius as -

$$\theta = \frac{\text{Arc length}}{\text{Radius}} \Rightarrow \theta = \frac{\overset{\frown}{AB}}{r}$$



If $r = 1$ then $\theta = AB$

The **radian measure** for a circle of unit radius of angle ABC is defined to be the length of the circular arc AB. since the circumference of the cirlce is 2p and one complete revolution of a cicle is 360°, the relation between radians and degrees is given by the following equation.

π radians = 180°

ANGLE CONVERSION FORMULAS

$$1 \text{ degree} = \frac{\pi}{180°} \quad (\approx 0.02) \text{ radian}$$

Degrees to radians : multiply by $\frac{\pi}{180°}$

1 radian ≈ 57 degrees

Radians to degrees : multiply by $\frac{180°}{\pi}$

EXAMPLE 15

Covert 45° to radians :

$$45 \bullet \frac{\pi}{180} = \frac{\pi}{4} \text{ rad}$$

Convert $\frac{\pi}{6}$ rad to degrees :

$$\frac{\pi}{6} \bullet \frac{180}{\pi} = 30°$$

EXAMPLE 16

Convert 30° to radians :

Sol. $30° \times \frac{\pi}{180°} = \frac{\pi}{6}$ rad

EXAMPLE 17

Convert $\frac{\pi}{3}$ rad to degrees.

Sol. $\frac{\pi}{3} \times \frac{180}{\pi} = 60$

Standard values

(1) $30° = \frac{\pi}{6}$ rad (2) $45° = \frac{\pi}{4}$ rad

(3) $60° = \frac{\pi}{3}$ rad (4) $90° = \frac{\pi}{2}$ rad

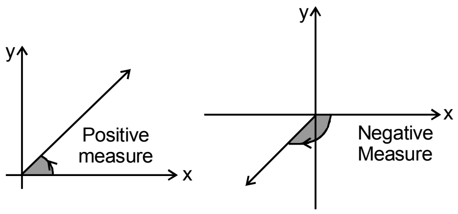
(5) $120° = \frac{2\pi}{3}$ rad (6) $135° = \frac{3\pi}{4}$ rad

(7) $150° = \frac{5\pi}{6}$ rad (8) $180° = \pi$ rad

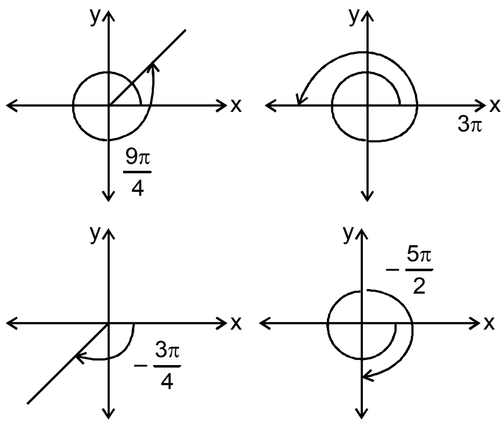
(9) $360° = 2\pi$ rad

(Check these values yourself to see that the satisfy the conversion formulaes)

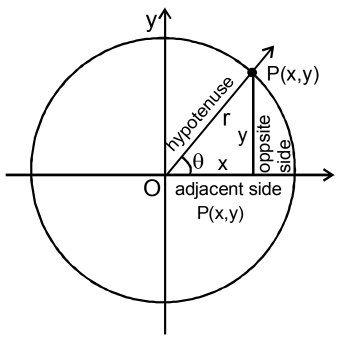
(ii) Measurement of positive & Negative Angles :



An angle in the xy-plane is said to be in standard position if its vertex lies at the origin and its initial ray lies along the positive x-axis (Fig). Angles measured counterclockwise from the positive x-axis are assigned positive measures; angles measured clockwise are assigned negative measures.



(iii) Six Basic Trigonometric Functions :



The trigonometric fucntion of a general angle θ are defined in tems of x, y and r.

Sine : $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$

Cosecant : $\operatorname{cosec}\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$

Cosine: $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$

Secant : $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$

Tangent: $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$

Cotangent: $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$

VALUES OF TRIGONOMETRIC FUNCTIONS

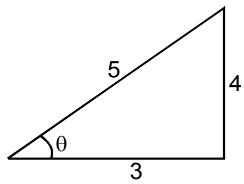
If the circle in (Fig. above) has radius r = 1, the equations defining sin θ and cos θ become

$\cos\theta = x, \qquad \sin\theta = y$

We can then calculate the values of the cosine and sine directly from the coordinates of P.

EXAMPLE 18

Find the six trigonometric ratios from given fig. (see above)



Sol. $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$ $\qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$

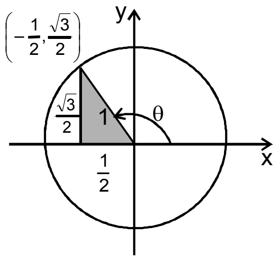
$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

$\sec\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$ $\qquad \operatorname{cosec}\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

EXAMPLE 19

Find the sine and cosine of angle θ shown in the unit circle if coordinate of point p are as shown.

Sol.



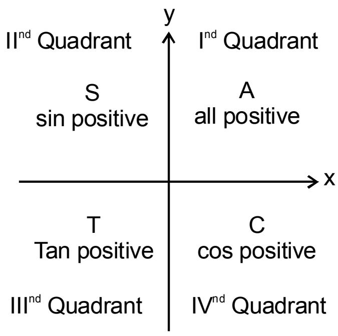
$\cos\theta$ = x-coordinate of P = $-\frac{1}{2}$

$\sin\theta$ = y-coordinate of P = $\frac{\sqrt{3}}{2}$

12. Values of sin θ, cos θ and tan θ for some standard angles.

Degree	0	30	37	45	53	60	90	120	135	180
Radians	0	$\pi/6$	$37\pi/180$	$\pi/4$	$53\pi/180$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\sin\theta$	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	0
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0	–1/2	$-1/\sqrt{2}$	–1
$\tan\theta$	0	$1/\sqrt{3}$	3/4	1	4/3	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	0

A useful rule for remembering when the basic trigonometric funcions are positive and negative is the CAST rule. If you are not very enthusiastic about CAST. You can remember it as ASTC (After school to college)



The CAST rule

RULES FOR FINDING TRIGONOMETRIC RATIO OF ANGLES GREATER THAN 90°.

Step 1 → Identify the quadrant in which angle lies.

Step 2 →

(a) If angle = $(n\pi \pm \theta)$ where n is an integer. Then

(b) If angle = $\left[(2n+1)\frac{\pi}{2}+\theta\right]$ where n is in interger. Then trigonometric function of $\left[(2n+1)\frac{\pi}{2}\pm\theta\right]$ = complimentry trignometric function of θ and sign will be decided by CAST Rule.

EXAMPLE 20

Evaluate sin 120°

Sol. $\sin 120° = \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

Aliter $\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

EXAMPLE 21

Evaluate cos 210°

Sol. $\cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

EXAMPLE 22

$$\tan 210° = \tan(180° + 30°) = \tan 30° = +\frac{1}{\sqrt{3}}$$

13. IMPORTANT FORMULAS

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

(iv) $\sin 2\theta = 2\sin\theta\cos\theta$

(v) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$= \cos^2\theta - \sin^2\theta$

(vi) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(vii) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(viii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ix) $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

(x) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(xi) $\cos C - \cos D = 2\sin\frac{D-C}{2}\sin\frac{C+D}{2}$

(xii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(xiii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(xiv) $\sin(90° + \theta) = \cos\theta$

(xv) $\cos(90° + \theta) = -\sin\theta$

(xvi) $\tan(90° + \theta) = -\cot\theta$

(xvii) $\sin(90° - \theta) = \cos\theta$

(xviii) $\cos(90° - \theta) = \sin\theta$

(xix) $\cos(180° - \theta) = -\cos\theta$

(xx) $\sin(180° - \theta) = \sin\theta$

(xxi) $\cos(180° + \theta) = -\cos\theta$

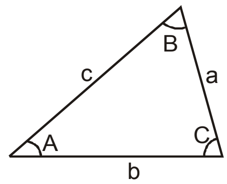
(xxii) $\tan(180° + \theta) = \tan\theta$

(xxiii) $\sin(-\theta) = -\sin\theta$

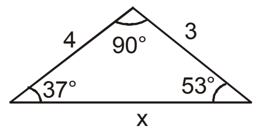
(xxiv) $\cos(-\theta) = \cos\theta$

(xxv) $\tan(-\theta) = -\tan\theta$

- **Sine Rule** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- **Cosine rule** $a^2 = b^2 + c^2 - 2bc\cos A$



EXAMPLE 23



Find x :

Sol. $\frac{\sin 90°}{x} = \frac{\sin 53°}{4}$

$x = 5$

14. SMALL ANGLE APPROXIMATION

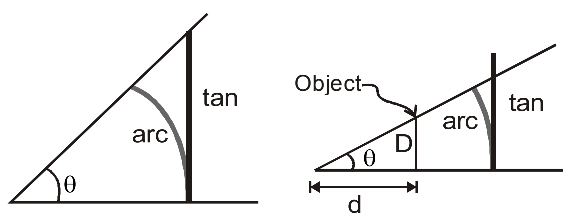
It is a useful simplification which is only approximately true for finite angles. It involves linerarization of the trigonometric functions so that, when the angle θ is measured in radians.

$$\sin\theta \approx \theta$$

$\cos\theta \approx 1$ or $\cos\theta \approx 1 - \frac{\theta^2}{2}$ for the second - order approximation

$\tan\theta \approx \theta$

Geometric justification



Small angle approximation. The value of the small angle X in radians is approximately equal to its tangent