26th Feb. 2021 | Shift - 2
MATHEMATICS

JEE I NEET I Foundation

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SELECTIONS SINCE 2007

Topic :- 3D
Subtopic:- Line & Plane
Level :- Tough

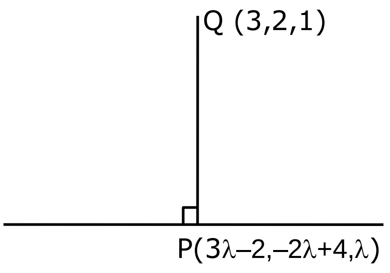
1. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :
(1) 142
(2) 68
(3) 136
(4) 102

माना समतलों $x + 2y + z = 6$ तथा $y + 2z = 4$ के प्रतिच्छेदन प्राप्त रेखा L है। यदि (3, 2, 1) से रेखा L पर लम्ब का पाद बिंदु $P(\alpha, \beta, \gamma)$ है, तो $21(\alpha + \beta + \gamma)$ का मान बराबर है :
(1) 142
(2) 68
(3) 136
(4) 102

Ans. (4)

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

Dr/s :- (3,–2, 1)
Points on the line (–2,4,0)
Equation of the line $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z}{1} = \lambda$



Dr's of PQ ; $3\lambda-5, -2\lambda + 2.\ \lambda-1$
Dr's of y lines are (3, –2, 1)
Since PQ ⊥ line
$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$
$\lambda = \dfrac{10}{7}$

$P\left(\frac{16}{7},\frac{8}{7},\frac{10}{7}\right)$

$21(\alpha + \beta + \gamma) = 21\left(\frac{34}{7}\right) = 102$

Topic :- Progression
Subtopic:- Miscellaneous
Level :- Tough

2. The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(2) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(3) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

(4) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

श्रेणी $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ का योगफल बराबर है :

(1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(2) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(3) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

(4) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

Ans. (3)

Sol. $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$

Put 2n + 1 = r, where r = 3,5,7,...

$\Rightarrow n = \frac{r-1}{2}$

$$\frac{n^2-6n+10}{(2n+1)!} = \frac{\left(\frac{r-1}{2}\right)^2+3r-3+10}{r!} = \frac{r^2+10r+29}{4r!}$$

Now $\sum_{r=3,5,7} \frac{r(r-1)+11r+29}{4r!} = \frac{1}{4}\sum_{r=3,5,7,......}\left(\frac{1}{(r-2)!}+\frac{11}{(r-1)!}+\frac{29}{r!}\right)$

$= \frac{1}{4}\left\{\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+.....\right)+11\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....\right)+29\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+.....\right)\right\}$

$= \frac{1}{4}\left\{\frac{e-\frac{1}{e}}{2}+11\left(\frac{e+\frac{1}{e}-2}{2}\right)+29\left(\frac{e-\frac{1}{e}-2}{2}\right)\right\}$

$= \frac{1}{8}\left\{e-\frac{1}{e}+11e+\frac{11}{e}-22+29e-\frac{29}{e}-58\right\}$

$= \frac{1}{8}\left\{41e-\frac{19}{e}-80\right\}$

Topic :- MOD
Subtopic:- Mixed
Level :- Easy

3. Let f(x) be a differentiable function at x = a with f′(a) = 2 and f(a) = 4. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

(1) 2a + 4
(2) 2a – 4
(3) 4 – 2a
(4) a + 4

माना एक फलन f(x), x = a पर अवकलनीय है तथा f′(a) = 2 और f(a) = 4 हैं। तो $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ बराबर है :

(1) 2a + 4
(2) 2a – 4
(3) 4 – 2a
(4) a + 4

Ans. (3)

Sol. By L-H rule

$L = \lim_{x\to a}\frac{f(a)-af'(x)}{1}$

$\therefore$ L = 4 – 2a

Topic :- Circle
Subtopic:- Mixed
Level :- Medium

4. Let A (1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P, A and B lie on :

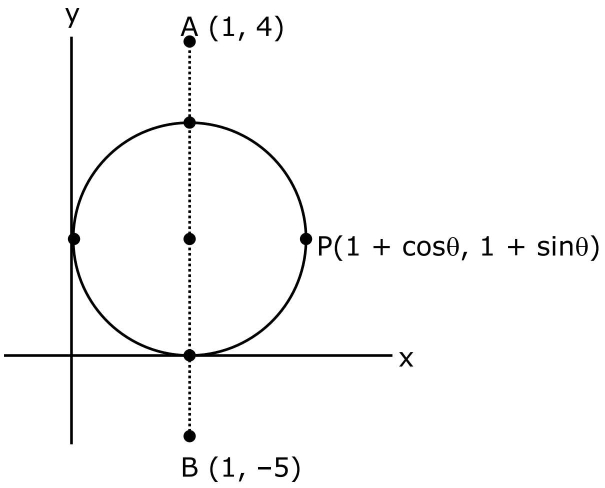
(1) a parabola
(2) a straight line
(3) a hyperbola
(4) an ellipse

माना दो बिंदु A (1, 4) तथा B(1, –5) हैं। माना वृत $(x-1)^2 + (y-1)^2 = 1$ पर P एक बिंदु है, जिसके लिए $(PA)^2+(PB)^2$ का मान अधिकतम है, तो बिंदु P, A तथा B निम्न में से किस पर स्थित है ?

(1) एक परवलय
(2) एक सरल रेखा
(3) एक अतिपरवलय
(4) एक दीर्घवृत

Ans. (2)

Sol.



$\therefore PA^2 = \cos^2\theta + (\sin\theta - 3)^2 = 10 - 6\sin\theta$

$PB^2 = \cos^2\theta + (\sin\theta - 6)^2 = 37 - 12\sin\theta$

$PA^2 + PB^2|_{max.} = 47 - 18\sin\theta|_{min.} \Rightarrow \theta = \frac{3\pi}{2}$

$\therefore$ P, A, B lie on a line x = 1

Topic :- Circle
Subtopic:- locus
Level :- Medium

5. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r, then r is equal to :

(1) $\frac{1}{4}$

(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{3}$

यदि बिंदु (3, 2) से वृत्त $x^2 + y^2 = 1$ के किसी बिंदु तक रेखा–खण्ड के मध्य–बिंदु का बिंदुपथ, r त्रिज्या का एक वृत है, तो r बराबर है :

(1) $\frac{1}{4}$

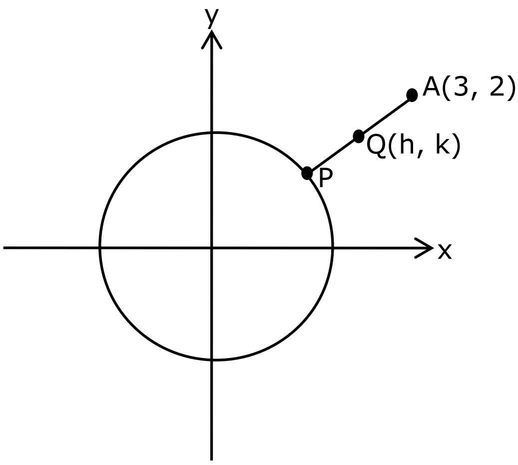
(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{3}$

Ans. (2)

Sol.



$\therefore$ P $\equiv$ (2h – 3, 2k – 2) →on circle

$\therefore \left(h - \frac{3}{2}\right)^2 + (k - 1)^2 = \frac{1}{4}$

$\Rightarrow$radius = $\frac{1}{2}$

Topic :- D.E.
Subtopic:- Special Case
Level :- Tough

6. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = – 2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $-\frac{18}{11}$

(2) $-\frac{18}{19}$

(3) $-\frac{4}{3}$

(4) $\frac{18}{35}$

माना एक वक्र के किसी बिंदु P(x, y) पर स्पर्श रेखा की प्रवणता $\frac{xy^2+y}{x}$ द्वारा दी गई है। यदि यह वक्र, रेखा x + 2y = 4 को x = – 2 पर काटता है, तो y का वह मान, जिसके लिए बिंदु (3, y) वक्र पर है, है :

(1) $-\frac{18}{11}$

(2) $-\frac{18}{19}$

(3) $-\frac{4}{3}$

(4) $\frac{18}{35}$

Ans. (2)

Sol. $\frac{dy}{dx}=\frac{xy^2+y}{x}$

$\Rightarrow \frac{xdy-ydx}{y^2}=xdx$

$\Rightarrow -d\left(\frac{x}{y}\right)=d\left(\frac{x^2}{2}\right)$

$\Rightarrow \frac{-x}{y}=\frac{x^2}{2}+C$

Curve intersect the line x + 2y = 4 at x = – 2

So, –2 + 2y = 4 $\Rightarrow$ y = 3

So the curve passes through (–2, 3)

$\Rightarrow \frac{2}{3}=2+C$

$\Rightarrow C = \frac{-4}{3}$

$\therefore$ curve is $\frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$

It also passes through (3, y)

$\frac{-3}{y} = \frac{9}{2} - \frac{4}{3}$

$\Rightarrow \frac{-3}{y} = \frac{19}{6}$

$\Rightarrow y = -\frac{18}{19}$

Topic :- A.U.C.
Subtopic:- Area between
Level :- Medium

7. Let A_1 be the area of the region bounded by the curves y = sinx, y = cos x and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sin x, y = cos x, x-axis and x= $\frac{\pi}{2}$ in the first quadrant. Then,

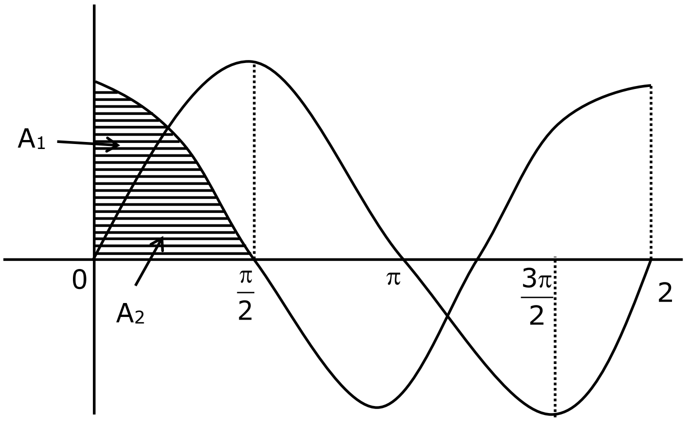
(1) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$
(2) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$
(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$
(4) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

माना वक्रों y = sinx, y = cos x तथा y-अक्ष द्वारा प्रथम चतुर्थांश में घिरे क्षेत्र का क्षेत्रफल A_1 है और माना वक्रों y = sin x, y = cos x, x-अक्ष तथा x= $\frac{\pi}{2}$ द्वारा प्रथम चतुर्थांश में घिरे क्षेत्र का क्षेत्रफल A_2 है। तो :

(1) $A_1 = A_2$ तथा $A_1 + A_2 = \sqrt{2}$
(2) $A_1 : A_2 = 1 : 2$ तथा $A_1 + A_2 = 1$
(3) $2A_1 = A_2$ तथा $A_1 + A_2 = 1 + \sqrt{2}$
(4) $A_1 : A_2 = 1 : \sqrt{2}$ तथा $A_1 + A_2 = 1$

Ans. (4)

Sol. $A_1 + A_2 = \int_0^{\pi/2} \cos x . dx = \sin x \Big|_0^{\pi/2} = 1$



$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx = (\sin x + \cos x)\Big|_0^{\pi/4} = \sqrt{2} - 1$$

$$\therefore A_2 = 1 - \left(\sqrt{2} - 1\right) = 2 - \sqrt{2}$$

$$\therefore \frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{\sqrt{2}\left(\sqrt{2} - 1\right)} = \frac{1}{\sqrt{2}}$$

Topic :- I.T.F.
Subtopic:- Mixed
Level :- Tough

8. If $0 < a, b < 1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + ..$ is :

(1) $\log_e 2$
(2) $\log_e\left(\frac{e}{2}\right)$
(3) e
(4) $e^2 - 1$

यदि $0 < a, b < 1$ तथा $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$ है, तो

$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + ..$ का मान है :

(1) $\log_e 2$
(2) $\log_e\left(\frac{e}{2}\right)$
(3) e
(4) $e^2 - 1$

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Ans. (1)

Sol. $\tan^{-1}\left(\frac{a+b}{1-ab}\right)=\frac{\pi}{4} \Rightarrow a + b = 1-ab \Rightarrow (1 + a)(1 + b) = 2$

Now, $(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)....\infty$

$=\left(a-\frac{a^2}{2}+\frac{a^3}{3}....\right)+\left(b-\frac{b^2}{2}+\frac{b^3}{3}....\right)$

$\log_e(1 + a) + \log_e(1 + b) = \log_e(1 + a)(1 + b) = \log_e 2$

Topic :- Jee mains topic
Subtopic:- Mathematical Reasoning
Level :- Tough

9. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then:

(1) F_1 is not a tautology but F_2 is a tautology
(2) F_1 is a tautology but F_2 is not a tautology
(3) F_1 and F_2 both are tautologies
(4) Both F_1 and F_2 are not tautologies

माना $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ तथा $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ दो तर्क संख्या व्यंजक है। तो :

(1) F_1 एक पुनरूक्ति नहीं है परन्तु F_2 एक पुनरूक्ति है
(2) F_1 एक पुनरूक्ति हैं परन्तु F_2 एक पुनरूक्ति नहीं है
(3) F_1 तथा F_2 दोंनों पुनरूक्ति है
(4) F_1 तथा F_2 दोनों पुनरूक्ति नहीं है

Ans. (1)

Sol. Truth table for F_1

A	B	C	~A	~B	~C	A ∨□ B	A∨B	~C∨ (A∨B)	[□ C∧ (A∨B)]∨~A	(A ∧□ B)∨ [~C∧ (A∨B)]∨□ A
T	T	T	F	F	F	F	T	F	F	F
T	T	F	F	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	F	T	T

Not a tautology

Truth table for F_2

A	B	$A \vee B$	$\sim A$	$B \to \sim A$	$(A \vee B) \vee (B \to \sim A)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

F_1 not shows tautology and F_2 shows tautology

Topic :- Determinant
Subtopic:- commas Rule
Level :- Tough

10. Consider the following system of equations:

x + 2y – 3z = a

2x + 6y – 11 z = b

x – 2y + 7z = c,

Where a, b and c are real constants. Then the system of equations:

(1) has a unique solution when 5a = 2b + c

(2) has infinite number of solutions when 5a = 2b +c

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

निम्न समीकरण निकाय पर विचार कीजिए

x + 2y – 3z = a

2x + 6y – 11 z = b

x – 2y + 7z = c,

जहाँ a, b तथा c वास्तविक अचर हैं। तो इस समीकरण निकाय :

(1) का केवल एक हल है जब 5a = 2b + c है

(2) के अनन्त हल हैं जब 5a = 2b +c है

(3) का सभी a, b तथा c के लिए कोई हल नहीं है

(4) का सभी a, b तथा c के लिए केवल एक हल है

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$

= 20 – 2(25) –3(–10)

= 20 – 50 + 30 = 0

$$D_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

= 20a – 2(7b + 11c) –3(–2b – 6c)
= 20a – 14b – 22c + 6b + 18c
= 20a – 8b – 4c
= 4(5a – 2b – c)

$$D_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

= 7b + 11c – a(25) – 3(2c – b)
= 7b + 11c – 25a – 6c + 3b
= – 25a + 10b + 5c
= –5(5a – 2b – c)

$$D_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

= 6c + 2b – 2(2c – b) – 10a
= – 10a + 4b + 2c
= –2(5a – 2b – c)
for infinite solution
$D = D_1 = D_2 = D_3 = 0$
$\Rightarrow 5a = 2b + c$

Topic :- Probability
Subtopic:-
Level :- Easy

11. A seven digit number is formed using digit 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$

(2) $\frac{4}{7}$

(3) $\frac{3}{7}$

(4) $\frac{1}{7}$

अंको 3, 3, 4, 4, 4, 5, 5 के प्रयोग से एक सात अंको की संख्या बनाई गई है। इस तरह बनाई गई संख्या के 2 से विभाजित होने की प्रायिकता है :

(1) $\frac{6}{7}$

(2) $\frac{4}{7}$

(3) $\frac{3}{7}$

(4) $\frac{1}{7}$

Ans. (3)

Sol. $n(s) = \frac{7!}{2!3!2!}$

$n(E) = \frac{6!}{2!2!2!}$

$P(E) = \frac{n(E)}{n(S)} = \frac{6!}{7!} \times \frac{2!3!2!}{2!2!2!}$

$\frac{1}{7} \times 3 = \frac{3}{7}$

Topic :- Vector
Subtopic:- Collinear Vactor
Level :- Medium

12. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

(1) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

(2) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(4) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

यदि दो सदिश $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ तथा $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ संरेख है, तो सदिश $x\hat{i} + y\hat{j} + z\hat{k}$ के समान्तर एक सम्भव इकाई सदिश है :

(1) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

(2) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

(4) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

Ans. (3)

Sol. $\frac{x}{1}=-\frac{1}{y}=\frac{1}{z}=\lambda(\text{let})$

Unit vector parallel to $x\hat{i}+y\hat{j}+z\hat{k}=\pm\frac{\left(\lambda\hat{i}-\frac{1}{\lambda}\hat{j}+\frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2+\frac{2}{\lambda^2}}}$

For $\lambda = 1$, it is $\pm\frac{(\hat{i}-\hat{j}+\hat{k})}{\sqrt{3}}$

Topic :- Definite Int.
Subtopic:- Substitution
Level :- Medium

13. For $x>0$, if $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to :

(1) $\frac{1}{2}$

(2) -1

(3) 1

(4) 0

$x>0$ के लिए यदि $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$ है, तो $f(e)+f\left(\frac{1}{e}\right)$ बराबर है :

(1) $\frac{1}{2}$

(2) -1

(3) 1

(4) 0

Ans. (1)

Sol. $f(e)+f\left(\frac{1}{e}\right)=\int_1^e \frac{\ell nt}{1+t}dt+\int_1^{1/e}\frac{\ell nt}{1+t}dt = I_1+I_2$

$I_2=\int_1^{1/e}\frac{\ell nt}{1+t}dt$ put $t=\frac{1}{z}, dt=-\frac{dz}{z^2}$

$=\int_1^e -\frac{\ell nz}{1+\frac{1}{z}}\times\left(-\frac{dz}{z^2}\right)=\int_1^e\frac{\ell nz}{z(z+1)}dz$

$f(e)+f\left(\frac{1}{e}\right)=\int_1^e\frac{\ell nt}{1+t}dt+\int_1^e\frac{\ell nt}{t(t+1)}dt=\int_1^e\frac{\ell nt}{1+t}+\frac{\ell nt}{t(t+1)}dt$

$=\int_1^e\frac{\ell nt}{t}dt\ \{\ln t=u, \frac{1}{t}dt\}$

$=du=\int_0^1 u\,du=\left.\frac{u^2}{2}\right|_0^1=\frac{1}{2}$

Topic :- Continuity

Subtopic:- Finding unknown consh

Level :- Medium

14. Let $f: R\to R$ be defined as

$$f(x)=\begin{cases}2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1\\ |ax^2+x+b|, & \text{if } -1\le x\le 1\\ \sin(\pi x) & \text{if } x>1\end{cases}$$

If f(x) is continuous on R, then a+b equals :

(1) 3 (2) −1 (3) −3 (4) 1

माना $f: R\to R$ निम्न द्वारा परिभाषित है :

$$f(x)=\begin{cases}2\sin\left(-\frac{\pi x}{2}\right), & \text{यदि } x<-1\\ |ax^2+x+b|, & \text{यदि } -1\le x\le 1\\ \sin(\pi x) & \text{यदि } x>1\end{cases}$$

यदि f(x), R पर संतत् है, तो a+b बराबर है :

(1) 3 (2) −1 (3) −3 (4) 1

Ans. (2)

Sol. If f is continuous at x = −1, then

$f(-1^-) = f(-1)$

$\Rightarrow 2 = |a - 1 + b|$

$\Rightarrow |a + b - 1| = 2$ ..... (i)

similarly

$f(1^-) = f(1)$

$\Rightarrow |a + b + 1| = 0$

$\Rightarrow a + b = -1$

Topic :- Function
Subtopic:- No. of function
Level :- Medium

15. Let A = {1,2,3......,10}and f: A→ A be defined as

$$f(k)=\begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$$

Then the number of possible functions g : A→A such that gof =f is :

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

माना A = {1,2,3......,10} है तथा f: A→ A

$$f(k)=\begin{cases} k+1 & \text{यदि } k \text{ विषम है} \\ k & \text{यदि } k \text{ सम है} \end{cases}$$

द्वारा परिभाषित है। तो ऐसे फलनों g : A → A जिनके लिए gof =f है, की सम्भावित संख्या है :

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

Ans. (1)

Sol. $g(f(x)) = f(x)$

$\Rightarrow g(x) = x$, when x is even.

5 elements in A can be mapped to any 10

So, $10^5 \times 1 = 10^5$

Topic :- Basic maths
Subtopic:- Mixed
Level :- Tough

16. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y+z=5$ and $y^{-1}+z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

(1) 11
(2) 6x
(3)12
(4) 6

एक प्राकृतिक संख्या के अभाज्य गुणनखंड n $=2^x3^y5^z$ द्वारा दिए गये हैं, जहाँ y तथा z के लिए y+z=5, $y^{-1}+z^{-1}=\frac{5}{6}$ तथा y > z हैं। तो n के विषम भाजकों की संख्या, जिनमें 1 भी है, है :

(1) 11
(2) 6x
(3)12
(4) 6

Ans. (3)

Sol. $y + z = 5$ ...(1)

$\frac{1}{y}+\frac{1}{z}=\frac{5}{6}$

$\Rightarrow \frac{y+z}{yz}=\frac{5}{6}$

$\Rightarrow \frac{5}{yz}=\frac{5}{6}$

$\Rightarrow yz = 6$

Also $(y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = 25 - 4(6) = 1$

$\Rightarrow y - z = 1$ ...(2)

from (1) and (2), y = 3 and z = 2

for calculating odd divisor of $p = 2^x.3^y.5^z$

x must be zero

$P = 2^0.3^3.5^2$

$\therefore$ total odd divisors must be $(3 + 1)(2 + 1) = 12$

Topic :- Function
Subtopic:- Composite Function
Level :- Medium

17. Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim\limits_{x\to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{4}{3},\infty\right)$

(2) $(-\infty, -1] \cup [2, \infty)$
(3) $(-\infty, -2] \cup [-1, \infty)$
(4) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

माना $f(x) = \sin^{-1} x$ तथा $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$ है। यदि $g(2) = \lim_{x\to 2} g(x)$, तो फलन fog का प्रांत है :

(1) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$
(2) $(-\infty, -1] \cup [2, \infty)$
(3) $(-\infty, -2] \cup [-1, \infty)$
(4) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

Ans. (1)

Sol. $g(2) = \lim_{x\to 2} \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$

For domain of fog (x)

$\left|\frac{x^2 - x - 2}{2x^2 - x - 6}\right| \le 1 \Rightarrow (x + 1)^2 \le (2x + 3)^2 \Rightarrow 3x^2 + 10x + 8 \ge 0$

$\Rightarrow (3x + 4)(x + 2) \ge 0$

$x \in (-\infty, -2] \cup \left(-\frac{4}{3}, \infty\right]$

Topic :- 3D
Subtopic:- Image of pt.
Level :- Medium

18. If the mirror image of the point (1,3,5) with respect to the plane $4x-5y+2z = 8$ is (α,β,γ), then $5(\alpha + \beta + \gamma)$equals:

(1) 47 (2) 39 (3) 43 (4) 41

यदि समतल $4x-5y+2z = 8$ के सापेक्ष बिंदु (1,3,5) का दर्पण प्रतिबिम्ब (α,β,γ) है, तो $5(\alpha + \beta + \gamma)$ बराबर है :

(1) 47 (2) 39 (3) 43 (4) 41

Ans. (1)

Sol. Image of (1, 3, 5) in the plane $4x - 5y + 2z = 8$ is (α, β, γ)

$\Rightarrow \frac{\alpha - 1}{4} = \frac{\beta - 3}{-5} = \frac{\gamma - 5}{2} = -2\,\frac{(4(1) - 5(3) + 2(5) - 8)}{4^2 + 5^2 + 2^2} = \frac{2}{5}$

$\therefore \alpha = 1 + 4\left(\frac{2}{5}\right) = \frac{13}{5}$

$\beta = 3 - 5\left(\frac{2}{5}\right) = 1 \;= \frac{5}{5}$

$\gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$

Thus, $5(\alpha + \beta + \gamma) = 5\left(\frac{13}{5} + \frac{5}{5} + \frac{29}{5}\right) = 47$

Topic :- D.E.
Subtopic:- Mixed
Level :- Medium

19. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then f(x) equals.

(1) $2e^{(e^x-1)} - 1$ (2) $e^{(e^x-1)}$ (3) $2e^{e^x} - 1$ (4) $e^{e^x} - 1$

माना $f(x) = \int_0^x e^t f(t)dt + e^x$ सभी $x \in R$ के लिए एक अवकलनीय फलन है तो f(x) बराबर है :

(1) $2e^{(e^x-1)} - 1$ (2) $e^{(e^x-1)}$ (3) $2e^{e^x} - 1$ (4) $e^{e^x} - 1$

Ans. **(1)**

Sol. Given, $f(x) = \int_0^x e^t f(t)dt + e^x$ ...(1)

Differentiating both sides w.r.t x

$f'(x) = e^x \cdot f(x) + e^x$ (Using Newton Leibnitz Theorem)

$\Rightarrow \frac{f'(x)}{f(x)+1} = e^x$

Integrating w.r.t x

$\int \frac{f'(x)}{f(x)+1}dx = \int e^x dx$

$\Rightarrow \ell n\,(f(x) + 1) = e^x + c$

Put x = 0

$\ell n\,2 = 1 + c$ $(\because f(0) = 1$, from equation (1))

$\therefore \ell n(f(x) + 1) = e^x + \ell n 2 - 1$

$\Rightarrow f(x) + 1 = 2.\; e^{e^x-1}$

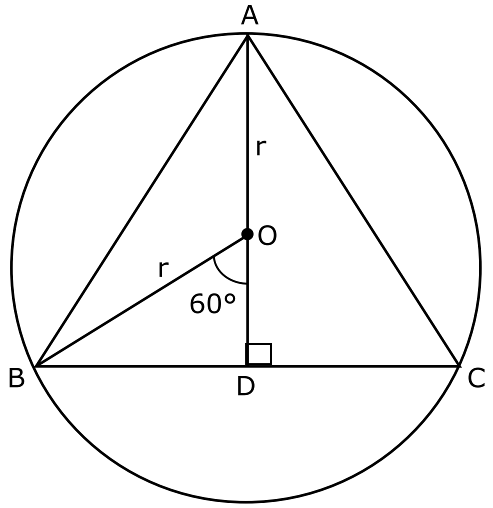
$\Rightarrow f(x) = 2e^{e^x-1} - 1$

Topic :- Maxima & minima
Subtopic:- Mixed
Level :- Easy

20. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is:
(1) A right angle triangle having two of its sides of length 2r and r.
(2) An equilateral triangle of height $\frac{2r}{3}$.
(3) An isosceles triangle with base equal to 2r.
(4) An equilateral triangle having each of its side of length $\sqrt{3}$ r.
'r' त्रिज्या के एक वृत के अंतर्गत अधिकतम क्षेत्रफल का त्रिभुज निम्न में से कौन सा है ?
(1) एक समकोण त्रिभुज जिसकी दो भुजाओं की लम्बाई 2r तथा r है
(2) एक समबाहु त्रिभुज जिसकी ऊँचाई $\frac{2r}{3}$ है
(3) एक समद्विबाहु त्रिभुज जिसका आधार 2r है
(4) एक समबाहु त्रिभुज जिसकी प्रत्येक भुजा की लम्बाई $\sqrt{3}$ r है

Ans. **(4)**

Sol. Triangle of maximum area that can be inscribed in a circle is an equilateral triangle.
Let ΔABC be inscribed in circle,



Now, in ΔOBD

$OD = r\cos 60° = \frac{r}{2}$

$\text{Height} = AD = \frac{3r}{2}$

Again in ΔABD

$\text{Now } \sin 60° = \frac{3\frac{r}{2}}{AB}$

$\Rightarrow AB = \sqrt{3}\, r$

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Section - B
Topic :- Progression
Subtopic:- Mixed
Level :- Medium

1. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is
4–अंको की संख्याओं, जिनका 18 के साथ महत्तम सर्वनिष्ठ भाजक 3 है, की कुल संख्या है ।

Ans. 1000

Sol. Since, required number has G.C.D with 18 as 3. It must be odd multiple of '3' but not a multiple of '9'.
(i) Now, 4-digit number which are odd multiple of '3' are,
1005,1011,1017, ... 9999 $\rightarrow$ 1499
(ii) 4–digit number which are odd multiple of 9 are,
1017, 1035, ... 9999 $\rightarrow$ 499
$\because$ Required numbers = 1499–499 = 1000

Topic :- Q.E.
Subtopic:- Sum & product
Level :- Medium

2. Let α and β be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta = -1$. Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of P_n^2 is________________.
माना α तथा β दो वास्तविक संख्याऐं हैं जिनके लिए $\alpha+\beta=1$ तथा $\alpha\beta = -1$ हैं। माना किसी पूर्णांक $n \geq 1$ के लिए $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ तथा $P_{n+1} = 29$ हैं। तो P_n^2 का मान है________________ ।

Ans. 324

Sol. Given, $\alpha + \beta = 1$, $\alpha\beta = -1$
$\therefore$ Quadratic equation with roots α,β is $x^2-x-1 = 0$
$\Rightarrow \alpha^2 = \alpha + 1$
Multiplying both sides by α^{n-1}
$\alpha^{n+1} = \alpha^n + \alpha^{n-1}$ ________ (1)
Similarly,
$\beta^{n+1} = \beta^n + \beta^{n-1}$ ________ (2)
Adding (1) & (2)
$\alpha^{n+1} + \beta^{n+1} = (\alpha^n+\beta^n) + (\alpha^{n-1}+\beta^{n-1})$
$\Rightarrow P_{n+1}=P_n+P_{n-1}$
$\Rightarrow 29 = P_n+11$ (Given, $P_{n+1} = 29$, $P_{n-1} = 11$)
$\Rightarrow P_n = 18$
$\therefore P_n^2 = 18^2 = 324$

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Topic :- Jee main topic
Subtopic:- Statistic
Level :- Tough

3. Let $X_1, X_2, \ldots\ldots\ldots X_{18}$ be eighteen observation such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is_______________.

माना $X_1, X_2, \ldots\ldots\ldots X_{18}$ अठारह प्रेक्षण हैं, जिनके लिए $\sum_{i=1}^{18}(X_i-\alpha)=36$ तथा $\sum_{i=1}^{18}(X_i-\beta)^2=90$ हैं, जहाँ α तथा β भिन्न वास्तविक संख्याऐं हैं। यदि इन प्रेक्षणों का मानक विचलन 1 हैं, तो $|\alpha - \beta|$ का मान बराबर है_______________।

Ans. 4

Sol. Given, $\sum_{i=1}^{18}(X_i-\alpha)=36$

$\Rightarrow \sum x_i - 18\alpha = 36$

$\Rightarrow \sum x_i - 18(\alpha+2)$ ...(1)

Also, $\sum_{i=1}^{18}(X_i-\beta)^2=90$

$\Rightarrow \sum x_i^2 + 18\beta^2 - 2\beta\sum x_i = 90$

$\Rightarrow \sum x_i^2 + 18\beta^2 + 2\beta \times 18(\alpha+2) = 90$ (using equation (1))

$\Rightarrow \sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

$\sigma^2 = 1 \Rightarrow \frac{1}{18}\sum x_i^2 - \left(\frac{\sum x_i}{18}\right)^2 = 1$ ($\because \sigma = 1$, given)

$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha+2)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$

$\Rightarrow \alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$

$\Rightarrow (\alpha-\beta)(\alpha-\beta+4) = 0$

$\Rightarrow \alpha - \beta = -4$

$\therefore |\alpha - \beta| = 4$ $(\alpha \neq \beta)$

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Topic :- D.I.
Subtopic:- King Theorem
Level :- Tough

4. In $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for m, n ≥ 1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}$, $\alpha\in R$, then α equals ______________ .

यदि $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n ≥ 1 तथा $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}$, $\alpha\in R$ है, तो α बराबर है ______________ ।

Ans. **1**

Sol. $I_{m,n}=\int_0^1 x^{m-1}.(1-x)^{n-1}dx$

Put $x=\frac{1}{y+1}\Rightarrow dx=\frac{-1}{(y+1)^2}dy$

$1-x=\frac{y}{y+1}$

$\therefore I_{m,n}=\int_\infty^0 \frac{y^{n-1}}{(y+1)^{m+n}}(-1)dy=\int_0^\infty \frac{y^{n-1}}{(y+1)^{m+n}}dy$ ...(i)

Similarly $I_{m,n}=\int_0^1 x^{n-1}.(1-x)^{m-1}dx$

$\Rightarrow I_{m,n}=\int_0^\infty \frac{y^{m-1}}{(y+1)^{m+n}}dy$ ...(ii)

From (i) & (ii)

$2I_{m,n}=\int_0^\infty \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy$

$\Rightarrow 2I_{m,n}=\underbrace{\int_0^1 \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy}_{I_1}+\underbrace{\int_1^\infty \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy}_{I_2}$

Put $y=\frac{1}{z}$ in I_2

$dy=-\frac{1}{z^2}dz$

$\Rightarrow 2I_{m,n}=\int_0^1 \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy+\int_1^0 \frac{z^{m-1}+z^{n-1}}{(z+1)^{m+n}}(-dz)$

$$\Rightarrow I_{m,n} = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy \Rightarrow \alpha = 1$$

Topic :- Ellipse
Subtopic:- Mixed
Level :- Medium

5. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _______________.

माना वक्रों $4x^2 + 9y^2 = 36$ तथा $(2x)^2 + (2y)^2 = 31$ की एक ऊभयनिष्ठ स्पर्श रेखा L है। तो रेखा L की प्रवणता का वर्ग बराबर है _______________ ।

Ans. **3**

Sol. $E : \frac{x^2}{9} + \frac{y^2}{4} = 1$ $\qquad C : x^2 + y^2 = \frac{31}{4}$

equation of tangent to ellipse is

$y = mx \pm \sqrt{9m^2 + 4}$ ...(i)

equation of tangent to circle is

$y = mx \pm \sqrt{\frac{31}{4}m^2 + \frac{31}{4}}$ ...(ii)

Comparing equation (i) & (ii)

$9m^2 + 4 = \frac{31}{4}m^2 + \frac{31}{4}$

$\Rightarrow 36m^2 + 16 = 31m^2 + 31$

$\Rightarrow 5m^2 = 15$

$\Rightarrow m^2 = 3$

Topic :- Matrix
Subtopic:- matrix Multiplication
Level :- Medium

6. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to_______________.

यदि किसी वास्तविक संख्याओं α तथा β के लिए आव्यूह $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$, समीकरण $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ को सन्तुष्ट करता है, तो $\beta - \alpha$ बराबर है_______________ ।

Ans. **4**

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

.
.
.
.
.

$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}, A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$L.H.S = A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20} + \alpha 2^{19} + 2\beta & 0 \\ 3\alpha + 3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$

$R.H.S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \alpha + \beta = 0$ and $2^{20} + \alpha 2^{19} + 2\beta = 4$

$\Rightarrow 2^{20} + \alpha (2^{19} - 2) = 4$

$\Rightarrow \alpha = \dfrac{4 - 2^{20}}{2^{19} - 2} = -2$

$\Rightarrow \beta = 2$

$\therefore \beta - \alpha = 4$

Topic :- Progression
Subtopic:- Mixed
Level :- Tough

7. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ………… satisfy the equation $4x^2 - 9x + 5 = 0$, then p+q is equal to ______________.

यदि अनुक्रम −16,8,−4,2,... के $p^{वें}$ तथा $q^{वें}$ पदों के समांतर माध्य तथा गुणोत्तर माध्य, समीकरण $4x^2-9x+5 = 0$ को संतुष्ट करते हैं, तो p+q बराबर है ______________।

Ans. 10

Sol. Given, $4x^2 - 9x + 5 = 0$

$\Rightarrow (x - 1)(4x - 5) = 0$

$\Rightarrow$ A.M $= \frac{5}{4}$, G.M = 1 $\qquad (\because \text{A.M} > \text{G.M})$

Again, for the series

−16, 8, −4, 2 ......

p^{th} term $t_p = -16\left(\frac{-1}{2}\right)^{p-1}$

q^{th} term $t_p = -16\left(\frac{-1}{2}\right)^{q-1}$

Now, A.M $= \frac{t_p + t_q}{2} = \frac{5}{4}$ & G.M $= \sqrt{t_p t_q} = 1$

$\Rightarrow 16^2\left(-\frac{1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2)^8 = (-2)^{(p+q-2)}$

$\Rightarrow p + q = 10$

Topic :- Tangent & normal

Subtopic:- normal of a cline

Level :- Tough

8. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, -3) and $(4, -2\sqrt{2})$, and given that $a - 2\sqrt{2}\ b = 3$, then (a^2+b^2+ab) is equal to______________.

माना एक वक्र के प्रत्येक बिंदु का अभिलम्ब, बिंदु (a, b) से होकर जाते है। यदि यह वक्र बिंदुओं (3, -3) तथा $(4, -2\sqrt{2})$, से होकर जाता है, तथा $a - 2\sqrt{2}\ b = 3$ है, तो (a^2+b^2+ab) बराबर है______________।

Ans. 9

Sol. Let the equation of normal is $Y - y = -\frac{1}{m}(X - x)$, where, m = $\frac{dy}{dx}$

As it passes through (a, b)

$b - y = -\frac{1}{m}(a - x) = -\frac{dx}{dy}(a - x)$

$\Rightarrow (b - y)dy = (x - a)dx$

by $-\frac{y^2}{2} = \frac{x^2}{2} - ax + c$ ...(i)

It passes through $(3,-3)$ & $(4, -2\sqrt{2})$

$\therefore \quad -3b - \frac{9}{2} = \frac{9}{2} - 3a + c$

$\Rightarrow -6b - 9 = 9 - 6a + 2c$

$\Rightarrow 6a - 6b - 2c = 18$

$\Rightarrow 3a - 3b - c = 9$ ...(ii)

Also

$-2\sqrt{2}b - 4 = 8 - 4a + c$

$4a - 2\sqrt{2}\,b - c = 12$ ...(iii)

Also $a - 2\sqrt{2}\,b = 3$ ...(iv) (given)

(ii) – (iii) $\Rightarrow -a + (2\sqrt{2} - 3)b = -3$ ....(v)

(iv) + (v) $\Rightarrow b = 0, \quad a = 3$

$\therefore a^2 + b^2 + ab = 9$

Topic :- Complex Number
Subtopic:- Mixed
Level :- Medium

9. Let z be those complex number which satisfy

$|z+5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is______________.

माना z वह सभी सम्मिश्र संख्याऐं हैं, जो $|z+5| \le 4$ तथा $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$ को सन्तुष्ट करती है। यदि $|z+1|^2$ का अधिकतम मान $\alpha + \beta\sqrt{2}$ है, तो $(\alpha + \beta)$ का मान है______________।

Ans. **48**

Sol. Given, $|z + 5| \le 4$

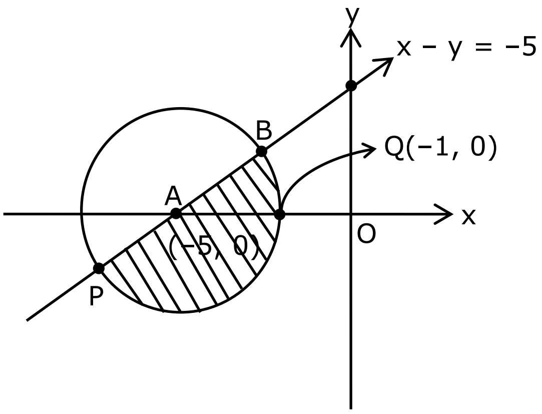
$\Rightarrow (x + 5)^2 + y^2 \le 16$ ...(1)

Also, $z(1+i) + \bar{z}(1-i) \ge -10$.

$\Rightarrow x - y \ge -5$ ...(2)

From (1) and (2)

Locus of z is the shaded region in the diagram.



|z + 1| represents distance of 'z' from Q(−1, 0)

Clearly 'p' is the required position of 'z' when |z + 1| is maximum.

$\therefore P \equiv \left(-5-2\sqrt{2}, -2\sqrt{2}\right)$

$\therefore (PQ)^2\Big|_{max} = 32 + 16\sqrt{2}$

$\Rightarrow \alpha = 32$

$\Rightarrow \beta = 16$

Thus, $\alpha + \beta = 48$

Topic :- Monotonocity

Subtopic:- Monotonocity increasing & decreasing

Level :- Medium

10. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval (a, a + 1).

Then, |a| is equal to_______________.

माना a एक पूर्णांक है जिसके लिए बहुपद $2x^5+5x^4+10x^3+10x^2+10x+10$ के सभी वास्तविक मूल अन्तराल (a, a + 1) में हैं। तो |a| बराबर है_______________।

Ans. **2**

Sol. Let, $f(x) = 2x^5+5x^4+10x^3+10x^2+10x+10$

$\Rightarrow f'(x) = 10\,(x^4 + 2x^3 + 3x^2 + 2x + 1)$

$= 10\left(x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) + 3\right)$

$= 10\left(\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) + 1\right)$

$= 10\left(\left(x + \frac{1}{x}\right) + 1\right)^2 > 0; \forall x \in R$

$\therefore$ f(x) is strictly increasing function. Since it is an odd degree polynomial it will have exactly one real root.

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Now, by observation

$f(-1) = 3 > 0$

$f(-2) = -64 + 80 - 80 + 40 - 20 + 10$

$= -34 < 0$

$\Rightarrow f(x)$ has at least one root in $(-2,-1) \equiv (a, a + 1)$

$\Rightarrow a = -2$

$\Rightarrow |a| = 2$

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