25th Feb. 2021 | Shift - 2
MATHEMATICS

JEE I NEET I Foundation

MOTION™

25000+
SELECTIONS SINCE 2007

SECTION-A

Topic :- 3D

Subtopic:- (M170)

Level :- Tough

1. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length:

(1) $\sqrt{\frac{2}{5}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{7}}$

1. एक समतल, बिन्दुओं A(1, 2, 3), B(2, 3, 1) तथा C(2, 4, 2) से होकर जाता है। यदि O मूल बिन्दु है तथा P, बिन्दु (2, –1, 1) है, तो इस समतल पर $\overrightarrow{OP}$ के प्रक्षेप की लम्बाई है:

(1) $\sqrt{\frac{2}{5}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{7}}$

Ans. (3)

Sol. A(1, 2, 3), B(2, 3, 1), C(2, 4, 2), O(0, 0, 0)

Equation of plane passing through A, B, C will be

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 2-1 & 4-2 & 2-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

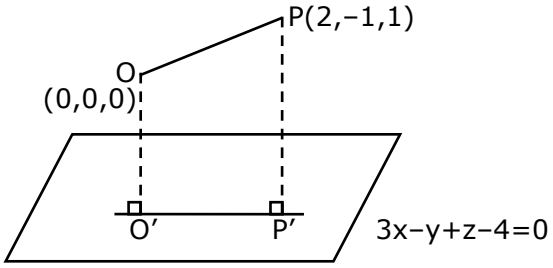
$\Rightarrow (x - 1)(-1 + 4) - (y - 2)(-1 + 2) + (z - 3)(2 - 1) = 0$

$\Rightarrow (x - 1)(3) - (y - 2)(1) + (z - 3)(1) = 0$

$\Rightarrow 3x - 3 - y + 2 + z - 3 = 0$

$\Rightarrow 3x - y + z - 4 = 0$, is the required plane.

Now, given O(0, 0, 0) & P(2, –1, 1)



Plane is 3x – y + z – 4 = 0

O' & P' are foot of perpendiculars.

for O'

$$\frac{x-0}{3}=\frac{y-0}{-1}=\frac{z-0}{1}=\frac{-(0-0+0-4)}{9+1+1}$$

$$\frac{x}{3}=\frac{y}{-1}=\frac{z}{1}=\frac{4}{11}$$

$$\Rightarrow \quad O'\left(\frac{12}{11},\frac{-4}{11},\frac{4}{11}\right)$$

for P'

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\frac{-(3(2)-(-1)+1-4)}{9+1+1}$$

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\left(\frac{-4}{11}\right)$$

$$P'\left(\frac{-12}{11}+2,\ \frac{4}{11}-1,\ \frac{-4}{11}+1\right)$$

$$\Rightarrow \quad P'\left(\frac{10}{11},\frac{-7}{11},\frac{7}{11}\right)$$

$$O'P'=\sqrt{\left(\frac{10}{11}-\frac{12}{11}\right)^2+\left(\frac{-7}{11}+\frac{4}{11}\right)^2+\left(\frac{7}{11}-\frac{4}{11}\right)^2}$$

$$\Rightarrow \quad O'P'=\frac{1}{11}\sqrt{4+9+9}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{22}}{11}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{2}\times\sqrt{11}}{11}$$

$$\Rightarrow \quad O'P'=\sqrt{\frac{2}{11}}$$

Topic :- Set & Relation

Subtopic:- Set & Relation

Level :- Medium

2. The contrapositive of the statement "If you will work, you will earn money" is:
(1) If you will not earn money, you will not work
(2) You will earn money, if you will not work
(3) If you will earn money, you will work
(4) To earn money, you need to work

2. कथन "यदि तुम काम करोगे, तुम धन कमाओगे" का प्रतिधनात्मक कथन है:
(1) यदि तुम धन नहीं कमाओगे, तुम काम नहीं करोगे
(2) तुम धन कमाओगे, यदि तुम काम नहीं करोगे
(3) यदि तुम धन कमाओगे, तुम काम करोगे
(4) धन कमाने के लिए, तुम्हें काम करने की आवश्यकता है

Ans. **(1)**

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
$p \rightarrow$ you will work
$q \rightarrow$ you will earn money
$\sim q \rightarrow$ you will not earn money
$\sim p \rightarrow$ you will not work
$\sim q \rightarrow \sim p \Rightarrow$ if you will not earn money, you will not work.

Topic :- Complex Number
Subtopic:- (M253)
Level :- Easy

3. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:
(1) 7 (2) −3 (3) 3 (4) −7

3. यदि $\alpha, \beta \in R$ हैं, जिनके लिए $z^2 + \alpha z + \beta = 0$ का एक मूल $1 - 2i$ (यहाँ $i^2 = -1$है) हैं, तो $(\alpha - \beta)$ बराबर है :
(1)7 (2) −3 (3) 3 (4) −7

Ans. **(4)**

Sol. $(1-2i)^2 + \alpha(1-2i) + \beta = 0$
$1 - 4 - 4i + \alpha - 2i\alpha + \beta = 0$
$(\alpha + \beta - 3) - i(4 + 2\alpha) = 0$
$\alpha + \beta - 3 = 0 \quad \& \quad 4 + 2\alpha = 0$
$\alpha = -2 \qquad \beta = 5$
$\alpha - \beta = -7$

Topic :- D.I.

Subtopic:- (M108)

Level :- Medium

4. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx$, then:

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(3) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.

(4) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.

4. यदि $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx$ है, तो :

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ G.P. में हैं

(2) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ A.P. में हैं

(3) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ A.P. में हैं

(4) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ G.P. में हैं

Ans. (2)

Sol. $$I_{n+2} + I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x.\cos ec^2 x\,dx = \left[\frac{-(\cot x)^{n+1}}{n+1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$I_{n+2} + I_n = \frac{1}{n+1}$$

$$I_2 + I_4 = \frac{1}{3}, I_3 + I_5 = \frac{1}{4}, I_4 + I_6 = \frac{1}{5}$$

Topic :- Matrix

Subtopic:- (M181)

Level :- Easy

5. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is:

(1) 1 (2) 3 (3) 2 (4)4

5. यदि आव्यूह $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$ के लिए $AA^T = I_2$ है तो $\alpha^4 + \beta^4$ का मान है :

(1) 1 (2) 3 (3) 2 (4) 4

Ans. (1)

Sol. $\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$1+\alpha^2 = 1$

$\alpha^2 = 0$

$\alpha^2+\beta^2 = 1$

$\beta^2 = 1$

$\alpha^4 = 0$

$\beta^4 = 1$

$\alpha^4+\beta^4 = 1$

Topic :- Function

Subtopic:- (M196)

Level :- Easy

6. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then:

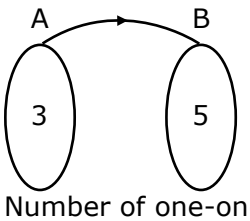
(1) y = 273x (2) 2y = 91x (3)y = 91x (4)2y = 273x

6. माना x ,3 अवयवों के एक समुच्य A से 5 अवयवों के एक समुच्चय B में एकैकी फलनों की कुल संख्या को दर्शाता है तथा y, समुच्चय A से समुच्चय A × B में एकैकी फलनों की कुल संख्या को दर्शाता है। तोः

(1)y = 273x (2) 2y = 91x (3)y = 91x (4)2y = 273x

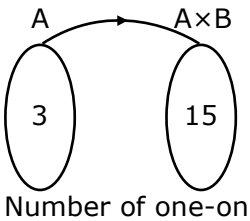
Ans. (2)

Sol. Number of elements in A = 3
Number of elements in B = 5
Number of elements in A × B = 15



Number of one-one function
$x = 5 \times 4 \times 3$

$x = 60$



Number of one-one function
$y = 15 \times 14 \times 13$

$y = 15 \times 4 \times \frac{14}{4} \times 13$

$y = 60 \times \frac{7}{2} \times 13$

$2y = (13)(7x)$
$2y = 91x$

Topic :- Ellipse

Subtopic:- (M280)

Level :- Medium

7. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

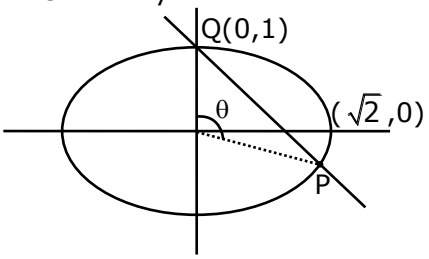
7. यदि वक्र, $x^2 + 2y^2 = 2$ रेखा $x + y = 1$ को दो बिन्दुओं P तथा Q पर काटता है, तो रेखा खंड PQ द्वारा मूल बिन्दु पर बनाया गया कोण है :

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Ans. (1)

Sol. Ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$

Line $x + y = 1$



Using homogenisation

$x^2 + 2y^2 = 2(1)^2$

$x^2 + 2y^2 = 2(x + y)^2$

$x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$x^2 + 4xy = 0$

for $ax^2 + 2hxy + by^2 = 0$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$\tan\theta = \left|\frac{2\sqrt{(2)^2 - 0}}{1 + 0}\right|$$

$\tan\theta = -4$

$\cot\theta = -\frac{1}{4}$

$\theta = \cot^{-1}\left(-\frac{1}{4}\right)$

$\theta = \pi - \cot^{-1}\left(\frac{1}{4}\right)$

$\theta = \pi - \left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)\right)$

$\theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Topic :- I.I.

Subtopic:- (M121)

Level :- Medium

8. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, $x > 0$, is equal to:

(where c is a constant of integration)

(1) $\log_e|x^2 + 5x - 7| + c$ (2) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$

(3) $4\log_e|x^2 + 5x - 7| + c$ (4) $\log_e\sqrt{x^2 + 5x - 7} + c$

8. समाकलन $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, $x > 0$, बराबर है :

(जहाँ c एक समाकलन अचर है)

(1) $\log_e|x^2 + 5x - 7| + c$ (2) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$

(3) $4\log_e|x^2 + 5x - 7| + c$ (4) $\log_e\sqrt{x^2 + 5x - 7} + c$

Ans. (3)

Sol. $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$

$= \int \frac{8x^3 + 5(4x^2)}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x + 20}{x^2 + 5x - 7}$

$= \int \frac{4(2x + 5)}{x^2 + 5x - 7}$ $\left\{\begin{matrix}\text{Let } x^2 + 5x - 7 = t \\ (2x + 5)dx = dt\end{matrix}\right\}$

$= \int \frac{4dt}{t}$

$= 4\ln|t| + C$

$= 4\ln\left|\left(x^2 + 5x - 7\right)\right| + c$

Topic :- Hyperbola
Subtopic:- (M273)
Level :- Medium

9. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

(1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

9. एक अतिपरवलय, दीर्घवृत्त $\frac{x^2}{25} + \frac{y^2}{16} = 1$ की नाभियों से होकर जाता है तथा इसके अनुप्रस्थ और संयुग्मी अक्ष क्रम ः दीर्घवृत्त के दीर्घ और अल्प अक्षों के समरूप हैं। यदि उनकी उत्केन्द्रताओं का गुणनफल एक है, तो अतिपरवलय का समीकरण हैः

(1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Ans. **(2)**

$e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$ foci (±ae, 0)

Foci $= (\pm 3, 0)$

Let equation of hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

Passes through (±3, 0)

Sol. $A^2 = 9, A = 3, e_2 = \frac{5}{3}$

$e_2^2 = 1 + \frac{B^2}{A^2}$

$\frac{25}{9} = 1 + \frac{B^2}{9} \Rightarrow B^2 = 16$

Ans $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Topic :- Limit
Subtopic:- (M61)
Level :- Easy

10. $\lim\limits_{x\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.....+\frac{n}{(2n-1)^2}\right]$ is equal to:

(1) 1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

10. $\lim\limits_{x\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.....+\frac{n}{(2n-1)^2}\right]$ बराबर है:

(1) 1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans. (3)

Sol.
$$\lim_{x\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\lim_{x\to\infty}\sum_{r=0}^{n-1}\frac{n^2}{n^2\left(1+\frac{r}{n}\right)^2}=\int_0^1\frac{dx}{(1+x)^2}$$

$$=-\left[\frac{1}{1+x}\right]_0^1\Rightarrow-\left[\frac{1}{2}-1\right]=\frac{1}{2}$$

Topic :- Prob.
Subtopic:- (M231)
Level :- Medium

11. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$ (2) $\frac{8}{45}$ (3) $\frac{14}{45}$ (4) $\frac{28}{45}$

11. 400 व्यक्तियों के एक समूह में, 160 धूम्रपान करते हैं तथा मांसाहारी है; 100 धूम्रपान करते हैं तथा भाकाहारी हैं और भोश 140 धूम्रपान नहीं करते तथा भाकाहारी हैं। उनको छाती के एक विशेष विकार होने का संयोग क्रमशः 35%, 20% तथा 10% है। इस समूह में से एक व्यक्ति यादृच्छिक चुना जाता है तथा यह पाया जाता है कि उसमें छाती का विकार है। उस चुने व्यक्ति के धूम्रपान करने वाले तथा मांसाहारी होने की प्रयिकता है:

(1) $\frac{7}{45}$ (2) $\frac{8}{45}$ (3) $\frac{14}{45}$ (4) $\frac{28}{45}$

Ans. (4)

Sol. Based on Baye's theorem

$$\text{Probability} = \frac{\left(160\times\frac{35}{100}\right)}{\left(160\times\frac{35}{100}\right)+\left(100\times\frac{20}{100}\right)+\left(140\times\frac{10}{100}\right)}$$

$$= \frac{5600}{9000}$$

$$= \frac{28}{45}$$

Topic :- Determinant

Subtopic:- (M190)

Level :- Medium

12. The following system of linear equations

$3x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) does not have any solution

(2) has a unique solution

(3) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(4) has infinitely many solutions

12. रैखिक समीकरण निकाय

$3x + 3y + 2z = 9$
$3x + 2y + 2z = 9$
$x - y + 4z = 8$

(1) का कोई हल नहीं है
(2) का केवल एक हल है
(3) का एक हल (α, β, γ) है जो $\alpha + \beta^2 + \gamma^3 = 12$ को संतुश्ट करता है
(4) के अनन्त हल है

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20 \neq 0 \quad \therefore$ unique solution

$$\Delta_x = \begin{vmatrix} 9 & 3 & 2 \\ 9 & 2 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 2 & 9 & 2 \\ 3 & 9 & 2 \\ 1 & 8 & 4 \end{vmatrix} = -20$$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 9 \\ 3 & 2 & 9 \\ 1 & -1 & 8 \end{vmatrix} = -40$$

$\therefore \quad x = \frac{\Delta_x}{\Delta} = 0$

$y = \frac{\Delta_y}{\Delta} = 1$

$z = \frac{\Delta_z}{\Delta} = 2$

Unique solution: (0, 1, 2)

Topic :- S & P

Subtopic:- (M21)

Level :- Easy

13. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where a, $x \in R$ and $a > 0$, is equal to:

(1) $a + \frac{1}{a}$ (2) $a + 1$ (3) $2a$ (4) $2\sqrt{a}$

13. $f(x) = a^{a^x} + a^{1-a^x}$, जहाँ a, $x \in R$ तथा $a > 0$ हैं, का न्यूनतम मान बराबर है :

(1) $a + \frac{1}{a}$ (2) $a + 1$ (3) $2a$ (4) $2\sqrt{a}$

Ans. (4)

Sol. AM $\geq$ GM

$$\frac{a^{ax} + \frac{a}{a^{ax}}}{2} \geq \left(a^{ax} \cdot \frac{a}{a^{ax}}\right)^{1/2} \Rightarrow a^{ax} + a^{1-ax} \geq 2\sqrt{a}$$

Topic :- Function

Subtopic:- (M200)

Level :- Medium

14. A function f(x) is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots\ldots + f\left(\frac{39}{20}\right)$

is equal to:

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{39}{2}$ (4) $\frac{29}{2}$

14. एक फलन f(x), $f(x) = \frac{5^x}{5^x + 5}$, द्वारा दिया गया है, तो श्रेणी

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots\ldots + f\left(\frac{39}{20}\right)$

का योगफल बराबर है :

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{39}{2}$ (4) $\frac{29}{2}$

Ans. (3)

Sol.

$$f(x)=\frac{5^x}{5^x+5} \ldots.(i)$$

$$f(2-x)=\frac{5^{2-x}}{5^{2-x}+5}$$

$$f(2-x)=\frac{5}{5^x+5} \ldots\ldots(ii)$$

Adding equation (i) and (ii)

$$f(x)+f(2-x)=1$$

$$f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)=1$$

$$f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)=1$$

:

:

$$f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)=1$$

$$\text{and } f\left(\frac{20}{20}\right)=f(1)=\frac{1}{2}$$

$$\Rightarrow 19+\frac{1}{2} \Rightarrow \frac{39}{2}$$

Topic :- Q.E.

Subtopic:- (M8)

Level :- Easy

15. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

(1) 4 (2) 1 (3) 2 (4) 3

15. माना $x^2 - 6x - 2 = 0$ के मूल α तथा β हैं। यदि $n \geq 1$ के लिए $a_n = \alpha^n - \beta^n$ है तो $\frac{a_{10} - 2a_8}{3a_9}$ का मान है :

(1) 4 (2) 1 (3) 2 (4) 3

Ans. (3)

Sol. $x^2 - 6x - 2 = 0 \begin{cases} \alpha \\ \beta \end{cases}$ $\alpha + \beta = 6$, $\alpha\beta = -2$

and $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$

$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$

Now
$$\frac{a_{10}-2a_8}{3a_9}=\frac{\left(\alpha^{10}-\beta^{10}\right)-2\left(\alpha^8-\beta^8\right)}{3(\alpha^9-\beta^9)}$$
$$=\frac{\left(\alpha^{10}-2\alpha^8\right)-\left(\beta^{10}-2\beta^8\right)}{3(\alpha^9-\beta^9)}$$
$$=\frac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{3(\alpha^9-\beta^9)}$$
$$=\frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{3(\alpha^9-\beta^9)}=\frac{6\left(\alpha^9-\beta^9\right)}{3(\alpha^9-\beta^9)}=\frac{6}{3}=2$$

Topic :- Determinant

Subtopic:- (M188)

Level :- Tough

16. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to:
(1) 64 (2) 16 (3) 80 (4) 128

16. माना A एक 3 × 3 आव्यूह है तथा det(A) = 4 है। माना R_i, आव्यूह A की $i^{वी}$ पंक्ति को दर्शाता है। यदि 2A पर संक्रिया $R_2 \to 2R_2 + 5R_3$ के प्रयोग से आव्यूह B प्राप्त होता है, तो det(B) बराबर है :
(1) 64 (2) 16 (3) 80 (4) 128

Ans. (1)

Sol.
$$A=\begin{bmatrix}R_{11} & R_{12} & R_{13}\\ R_{21} & R_{22} & R_{23}\\ R_{31} & R_{32} & R_{33}\end{bmatrix}$$
$$2A=\begin{bmatrix}2R_{11} & 2R_{12} & 2R_{13}\\ 2R_{21} & 2R_{22} & 2R_{23}\\ 2R_{31} & 2R_{32} & 2R_{33}\end{bmatrix}$$
$$R_2 \to 2R_2+5R_3$$
$$B=\begin{bmatrix}2R_{11} & 2R_{12} & 2R_{13}\\ 4R_{21}+10R_{31} & 4R_{22}+10R_{32} & 4R_{23}+10R_{33}\\ 2R_{31} & 2R_{32} & 2R_{33}\end{bmatrix}$$
$$R_2 \to R_2-5R_3$$
$$B=\begin{bmatrix}2R_{11} & 2R_{12} & 2R_{13}\\ 4R_{21} & 4R_{22} & 4R_{23}\\ 2R_{31} & 2R_{32} & 2R_{33}\end{bmatrix}$$
$$|B|=\begin{vmatrix}2R_{11} & 2R_{12} & 2R_{13}\\ 4R_{21} & 4R_{22} & 4R_{23}\\ 2R_{31} & 2R_{32} & 2R_{33}\end{vmatrix}$$
$$|B|=2\times2\times4\begin{vmatrix}R_{11} & R_{12} & R_{13}\\ R_{21} & R_{22} & R_{23}\\ R_{31} & R_{32} & R_{33}\end{vmatrix}$$
$$=16\times4$$
$$=64$$

Topic :- Parabola

Subtopic:- (M280)

Level :- Medium

17. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is:

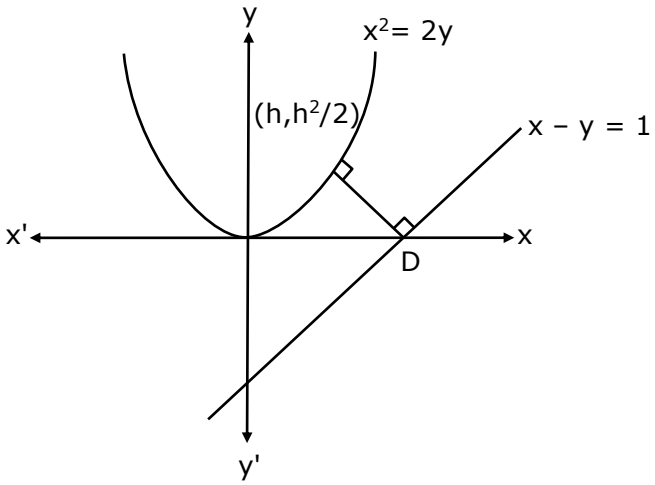
(1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

17. रेखा $x - y = 1$ तथा वक्र $x^2 = 2y$ के बीच की न्यूनतम दूरी है :

(1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. (3)

Sol. Shortest distance must be along common normal



m_1 (slope of line $x-y = 1$) $= 1 \Rightarrow$ slope of perpendicular line $= -1$

$m_2 = \frac{2x}{2} = x \Rightarrow m_2 = h \Rightarrow$ slope of normal $-\frac{1}{h}$

$-\frac{1}{h} = -1 \Rightarrow h = 1$

so point is $\left(1, \frac{1}{2}\right)$

$D = \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1+1}}\right| = \frac{1}{2\sqrt{2}}$

Topic :- Prob.

Subtopic:- (M234)

Level :- Tough

18. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is:

(1) $\frac{1}{5}$ (2) $\frac{2}{9}$ (3) $\frac{97}{297}$ (4) $\frac{122}{297}$

18. माना 4-अंकों की सभी धनपूर्णसंख्याओं, जिसका केवल एक अंक 7 है, का समुच्चय A है। तो A से यादृच्छिक चुने गये एक अवयव को 5 से विभाजित करने पर भोशफल 2 आने की प्रायिकता है :

(1) $\frac{1}{5}$ (2) $\frac{2}{9}$ (3) $\frac{97}{297}$ (4) $\frac{122}{297}$

Ans. (3)

Sol. Total cases

$(4 \times 9 \times 9 \times 9) - (3 \times 9 \times 9)$

$$\text{Probability} = \frac{(3\times9\times9)-(2\times9)+(8\times9\times9)}{(4\times9^3)-(3\times9^2)}$$

$$= \frac{97}{217}$$

Topic :- ITF

Subtopic:- (M128)

Level :- Medium

19. $\text{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

(1) $\frac{75}{56}$ (2) $\frac{65}{56}$ (3) $\frac{56}{33}$ (4) $\frac{65}{33}$

19. $\text{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ बराबर है:

(1) $\frac{75}{56}$ (2) $\frac{65}{56}$ (3) $\frac{56}{33}$ (4) $\frac{65}{33}$

Ans. (2)

Sol. $\text{cosec}\left(2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$\text{cosec}\left(2\tan^{-1}\left(\frac{1}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$$= \cos ec\left(\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} \right) + \cos^{-1}\left(\frac{4}{5}\right) \right)$$

$$= \cos ec\left(\tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{4}{5}\right) \right)$$

Let $\tan^{-1}(5/12) = \theta \Rightarrow \sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}$

and $\cos^{-1}\left(\frac{4}{5}\right) = \phi \Rightarrow \cos\phi = \frac{4}{5}$ and $\sin\phi = \frac{3}{5}$

$$= \cos ec(\theta + \phi)$$

$$= \frac{1}{\sin\theta\cos\phi + \cos\theta\sin\phi}$$

$$= \frac{1}{\frac{5}{13}.\frac{4}{5} + \frac{12}{13}.\frac{3}{5}} = \frac{65}{56}$$

Topic :- Trigo Phase - II

Subtopic:- Mixed (M38)

Level :- Medium

20. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to:

(1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{1-\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{2}$

20. यदि $0 < x, y < \pi$ तथा $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$ है, तो $\sin x + \cos y$ बराबर है :

(1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{1-\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{2}$

Ans. (1)

Sol.

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-\left[2\cos^2\left(\frac{x+y}{2}\right)-1\right]=\frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right]=\frac{1}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[2\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)\right]=\frac{1}{2}$$

$$\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)=\frac{1}{8}$$

Possible when $\frac{x}{2}=30°$ & $\frac{y}{2}=30°$

$x=y=60^{\circ}$

$$\sin x+\cos y=\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{\sqrt{3}+1}{2}$$

SECTION-B

Topic :- Limit

Subtopic:- (M61)

Level :- Medium

1. If $\lim\limits_{x\to0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of a – 2b is _____.

1. यदि $\lim\limits_{x\to0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ का अस्तित्व है तथा यह b के बराबर है, तो a – 2b का मान है _____ ।

Ans. (5)

$$\lim_{x\to0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$$

Applying L' Hospital Rule

$$\lim_{x\to0}\frac{a-4e^{4x}}{a\left(e^{4x}-1\right)+ax\left(4e^{4x}\right)}\quad \text{So } a=4$$

Sol. Applying L' Hospital Rule

$$\lim_{x\to0}\frac{-16e^{4x}}{a\left(4e^{4x}\right)+a\left(4e^{4x}\right)+ax\left(16e^{4x}\right)}$$

$$\frac{-16}{4a+4a}=\frac{-16}{32}=-\frac{1}{2}=b$$

$$a-2b=4-2\left(\frac{-1}{2}\right)=4+1=5$$

Topic :- Tangent & normal

Subtopic:- (M289)

Level :- Tough

2. $(x - 3)^2+y^2=9$and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a+c) is equal to _____.

2. वृत्त $(x - 3)^2+y^2=9$ तथा परवलय $y^2 = 4x$ की एक उभयनिश्ठ स्प र्ा रेखा है। यदि दो संपर्क बिन्दु (a, b) तथा (c, d) भिन्न हैं तथा प्रथम चतुर्थांश में है, तो 2(a+c) बराबर है _____ ।

Ans. (9)

Sol. Circle: $(x - 3)^2+ y^2= 9$

Parabola: $y^2 = 4x$

Let tangent $y = mx + \frac{a}{m}$

$$y = mx + \frac{1}{m}$$

$m^2x - my + 1 = 0$

the above line is also tangent to circle

$(x - 3)^2+ y^2= 9$

$\therefore \perp$ from (3, 0) = 3

$$\left|\frac{3m^2 - 0 + 1}{\sqrt{m^2 + m^4}}\right| = 3$$

$(3m^2 + 1)^2 = 9(m^2 + m^4)$

$6m^2 + 1 + 9m^4 = 9m^2 + 9m^4$

$3m^2 = 1$

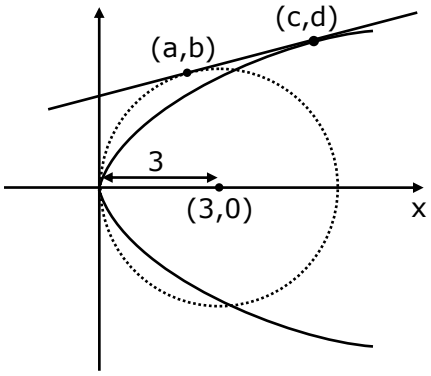
$$m = \pm\frac{1}{\sqrt{3}}$$

$\therefore$tangent is

$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ or $y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$

(it will be used) (rejected)

$$m = \frac{1}{\sqrt{3}}$$



for Parabola $\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv (3, 2\sqrt{3})$

(c, d)

for Circle $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ & $(x-3)^2 + y^2 = 9$

solving, $(x-3)^2 + \left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)^2 = 9$

$$x^2 + 9 - 6x + \frac{1}{3}x^2 + 3 + 2x = 9$$

$$\frac{4}{3}x^2 - 4x + 3 = 0$$

$$4x^2 - 12x + 9 = 0$$

$$4x^2 - 6x - 6x + 9 = 0$$

$$2x(2x-3) - 3(2x-3) = 0$$

$$(2x-3)(2x-3) = 0$$

$$x = \frac{3}{2}$$

$\therefore \quad y = \frac{1}{\sqrt{3}}\left(\frac{3}{2}\right) + \sqrt{3}$

$$y = \frac{\sqrt{3}}{2} + \sqrt{3}$$

$$y = \frac{3\sqrt{3}}{2}$$

$(a, b) \equiv \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$2(a + c) = 2\left(\frac{3}{2}+3\right)$

$= 2\left(\frac{3}{2}+\frac{6}{2}\right)$

$= 9$

Topic :- D.I.

Subtopic:- (M108)

Level :- Medium

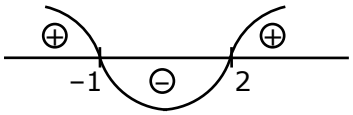
3. The value of $\int_{-2}^{2}\left|3x^2-3x-6\right|\,dx$ is _____.

3. $\int_{-2}^{2}\left|3x^2-3x-6\right|\,dx$ का मान है _____ ।

Ans. (19)

Sol. $3\int_{-2}^{2}\left|x^2-x-2\right|dx \qquad x^2-x-2$

$=(x-2)(x+1)$



$=3\left\{\int_{-2}^{-1}\left(x^2-x-2\right)dx+\int_{-1}^{2}\left(-x^2+x+2\right)dx\right\}$

$=3\left[\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-2}^{-1}-\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-1}^{2}\right]$

$=19$

Topic :- Binomial Theorem

Subtopic:- (M29)

Level :- Medium

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is _____.

4. यदि x को 4 से विभाजित करने पर भोशफल 3 है, तो $(2020+x)^{2022}$ को 8 विभाजित करने पर भोशफल है _____।

Ans. (1)

Sol. Let $x = 4k + 3$

$(2020 + x)^{2022}$

$= (2020 + 4k + 3)^{2022}$

$= (4(505) + 4k + 3)^{2022}$

$= (4P + 3)^{2022}$

$= (4P + 4 - 1)^{2022}$

$= (4A - 1)^{2022}$

$^{2022}C_0(4A)^0(-1)^{2022} + {}^{2022}C_1(4A)^1(-1)^{2021} + \ldots\ldots$

$1 + 8\lambda$

Reminder is 1.

Topic :- 3D

Subtopic:- (M168)

Level :- Tough

5. A line 'ℓ' passing through origin is perpendicular to the lines

$$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on 'ℓ_2' at the distance of $\sqrt{17}$ from the point of intersection of 'ℓ' and 'ℓ_1' are (a, b, c), then 18(a+b+c) is equal to _____.

5. मूल बिन्दु से होकर जानेवाली एक रेखा 'ℓ', रेखाओं

$$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

पर लम्बवत् है। यदि 'ℓ_2' पर प्रथम अश्टां ाक में एक बिन्दु (a, b, c) की 'ℓ' तथा 'ℓ_1' के प्रतिच्छेदन बिन्दु से दूरी $\sqrt{17}$ है, तो 18(a+b+c) बराबर है_____।

Ans. (44)

Sol. $\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$$\ell_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} \quad \Rightarrow \quad \text{D.R. of } \ell_1 = 1, 2, 2$$

$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

$$\ell_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} \quad \Rightarrow \quad \text{D.R. of } \ell_2 = 2, 2, 1$$

D.R. of ℓ is $\perp$ to $\ell_1 \& \ell_2$

$\therefore$ D.R. of ℓ || $(\ell_1 \times \ell_2)$ $\Rightarrow$ $\langle -2, 3, -2 \rangle$

$\therefore$ Equation of ℓ : $\frac{x}{2}=\frac{y}{-3}=\frac{z}{2}$

Solving ℓ &ℓ_1

$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + \mu)$

$\Rightarrow$ $2\lambda = \mu + 3$

$-3\lambda = 2\mu - 1$

$2\lambda = 2\mu + 4$

$\Rightarrow$ $\mu + 3 = 2\mu + 4$

$\mu = -1$

$\lambda = 1$

P(2, –3, 2) {intersection point}

Let, Q$(2\nu + 3, 2\nu + 3, \nu + 2)$ be point on ℓ_2

Now, PQ = $\sqrt{(2\nu+3-2)^2+(2\nu+3+3)^2+(\nu+2-2)^2}$ = $\sqrt{17}$

$\Rightarrow$ $(2\nu + 1)^2 + (2\nu + 6)^2 + (\nu)^2 = 17$

$\Rightarrow$ $9\nu^2 + 28\nu + 36 + 1 - 17 = 0$

$\Rightarrow$ $9\nu^2 + 28\nu + 20 = 0$

$\Rightarrow$ $9\nu^2 + 18\nu + 10\nu + 20 = 0$

$\Rightarrow$ $(9\nu + 10)(\nu + 2) = 0$

$\Rightarrow$ $\nu = -2$ (rejected), $-\frac{10}{9}$ (accepted)

$Q\left(3-\frac{20}{9}, 3-\frac{20}{9}, 2-\frac{10}{9}\right)$

$\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

$\therefore$ $18(a + b + c)$

$= 18\left(\frac{7}{9}+\frac{7}{9}+\frac{8}{9}\right)$

$= 44$

Topic :- Differentiability

Subtopic:- (M71)

Level :- Medium

6. A function f is defined on [–3,3] as

$$f(x)=\begin{cases}\min\{|x|, 2-x^2\}, & -2 \le x \le 2 \\ [|x|], & 2 < |x| \le 3\end{cases}$$

where [x] denotes the greatest integer $\le x$. The number of points, where f is not differentiable in (–3,3) is _____.

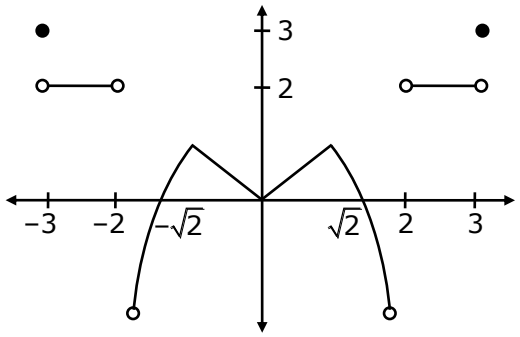
6. [−3,3] पर एक फलन f निम्न द्वारा परिभाषित है

$$f(x)=\begin{cases}\min\left\{|x|,2-x^2\right\}, & -2\le x\le 2\\ [|x|], & 2<|x|\le 3\end{cases}$$

जहाँ [x] महत्तम पूर्णांक $\le x$ है। (−3,3) में उन बिन्दुओं की संख्या, जहाँ f अवकलनीय नहीं है, है _____ ।

Ans. **(5)**

Sol.



Points of non-differentiability in (−3, 3) are at x = −2, −1, 0, 1, 2.

i.e. 5 points.

Topic :- Tangent & normal

Subtopic:- (M288)

Level :- Medium

7. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

7. यदि वक्र $x = y^4$ तथा $xy = k$ एक दूसरे को समकोण पर काटते हैं, तो $(4k)^6$ बराबर है _____ ।

Ans. **4**

Sol. $4y^3\dfrac{dy}{dx}=1$ & $x\dfrac{dy}{dx}+y=0$

$m_1=\dfrac{1}{4y^3}$ $\dfrac{dy}{dx}=\dfrac{-y}{x}=m_2$

$m_1m_2=-1$

$\dfrac{1}{4.y^3}\times\dfrac{-y}{x}=-1\ \because x=y^4$

$\dfrac{1}{4.y^6}=1$ and $xy=k$

$y^6=\dfrac{1}{4}$ $\Rightarrow k=y^5$

$\Rightarrow k^6=y^{30}$

$\Rightarrow k^6=\left(\dfrac{1}{4}\right)^5$

$\therefore (4k)^6 = 4^6 \times k^6 = 4$

Topic :- P & C

Subtopic:- (M223)

Level :- Medium

8. The total number of two digit numbers 'n', such that 3^n+7^n is a multiple of 10, is _____.

8. दो अंकों की संख्याओं 'n', जिनके लिए 3^n+7^n 10 का गुणज है, की कुल संख्या है _____.

Ans. (45)

Sol. $\because 7^n = (10-3)^n = 10K + (-3)^n$

$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$ → 10K if n = odd

→ $10K + 2.3^n$ if n = even

Let n = 2t; $t \in N$

$\therefore 3^n = 3^{2t} = (10-1)^t$

$= 10p + (-1)^t$

$= 10p \pm 1$

$\therefore$ if n = even then $7^n + 3^n$ will not be multiply of 10

So if n is odd then only $7^n + 3^n$ will be multiply of 10

$\therefore n = 11, 13, 15, ..., 99$

$\therefore$ Ans 45

Topic :- Vector

Subtopic:- (M161)

Level :- Easy

9. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to _____.

9. माना $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ तथा $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$ है। यदि समान्तर चर्तुभुज, जिसकी संलग्न भुजायें $\vec{a}$ तथा $\vec{b}$ हैं, का क्षेत्रफल $8\sqrt{3}$ वर्ग इकाई है, तो $\vec{a}\cdot\vec{b}$ बराबर है _____ ।

Ans. (2)

Sol. $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

Area of parallelogram = $|\vec{a} \times \vec{b}|$

$= \left|(\hat{i} + \alpha\hat{j} + 3\hat{k}) \times (3\hat{i} - \alpha\hat{j} + \hat{k})\right|$

$8\sqrt{3} = \left|(4\alpha)\hat{i} + 8\hat{j} - (4\alpha)\hat{k}\right|$

$(64)(3) = 16\alpha^2 + 64 + 16\alpha^2$

$(64)(3) = 32\alpha^2 + 64$

$6 = \alpha^2 + 2$

$\alpha^2 = 4$

$\therefore \quad \vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

$\vec{a}\cdot\vec{b} = 3 - \alpha^2 + 3$

$= 6 - \alpha^2$

$= 6 - 4$

$= 2$

Topic :- D.E.

Subtopic:- (M137)

Level :- Medium

10. If the curve y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdx = 0$, passes through the intersection of the lines, $2x - 3y=1$ and $3x+2y=8$, then $|y(1)|$ is equal to _____.

10. यदि अवकल समीकरण $(2xy^2 - y)dx + xdx = 0$ का हल y = y(x), रेखाओं $2x - 3y=1$ तथा $3x+2y=8$ के प्रतिच्छेदन बिन्दु से होकर जाता है तो $|y(1)|$ बराबर है _____ ।

Ans. 1

Sol. Given,

$(2xy^2 - y)dx + xdx = 0$

$\Rightarrow \quad \frac{dy}{dx} + 2y^2 - \frac{y}{x} = 0$

$\Rightarrow \quad -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{1}{x}\right) = 2$

$\frac{1}{y} = z$

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \quad \frac{dz}{dx} + z\left(\frac{1}{x}\right) = 2$

I.F. $= e^{\int \frac{1}{x}dx} = x$

$\therefore \quad z(x) = \int 2(x)dx = x^2 + c$

$\Rightarrow \quad \frac{x}{y} = x^2 + c$

As it passes through P(2, 1)

[Point of intersection of $2x - 3y = 1$ and $3x + 2y = 8$]

$\therefore \frac{2}{1} = 4 + c$

$\Rightarrow \quad c = -2$

$\Rightarrow \quad \frac{x}{y} = x^2 - 2$

Put x = 1

$\frac{1}{y} = 1 - 2 = -1$

$\Rightarrow \quad y(1) = -1$

$\Rightarrow \quad |y(1)| = 1$

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