25th Feb. 2021 | Shift - 1
MATHEMATICS

JEE I NEET I Foundation

MOTION™

25000+
SELECTIONS SINCE 2007

Section : Mathematics Section A

Topic :- Probabitity

Subtopic:- Mixed (M234)

Level :- Medium

1. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$ (2) $\frac{1}{72}$ (3) $\frac{1}{36}$ (4) $\frac{5}{216}$

द्विघातीय समीकरण $ax^2 + bx + c = 0$ के गुणाक a, b तथा c एक पासे को तीन बार उछाल कर प्राप्त किए जाते है। इस समीकरण के मूल बराबर होने की प्रायिकता हैं :

(1) $\frac{1}{54}$ (2) $\frac{1}{72}$ (3) $\frac{1}{36}$ (4) $\frac{5}{216}$

Ans. (4)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2$, $ac = 1$ $\Rightarrow$ $a = 1, c = 1$

If $b = 4$, $ac = 4$ $\Rightarrow$ $a = 1, c = 4$
$a = 4, c = 1$
$a = 2, c = 2$

If $b = 6$, $ac = 9 \Rightarrow$ $a = 3, c = 3$

$\therefore \text{probability} = \frac{5}{216}$

Topic :- 3D

Subtopic:- Direction cosines (M168)

Level :- Easy

2. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{5}{8}$ (4) $\frac{3}{8}$

माना दो रेखाऐं जिनकी दिक्कोज्यायें समीकरणों $l + m - n = 0$ तथा $l^2 + m^2 - n^2 = 0$ को सन्तुष्ट करती हैं, के बीच एक कोण α है। तो $\sin^4\alpha + \cos^4\alpha$ का मान है :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{5}{8}$ (4) $\frac{3}{8}$

Ans. (3)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore\ 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2}$ & $l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow l\ m = 0$ or $m = 0$

$\therefore l = 0, m = \frac{1}{\sqrt{2}}$ or $l = \frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

Topic :- Indefinite Integration

Subtopic:- Substitution Medium (M121)

Level :- Medium

3. The value of the integral

$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ is

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

समाकलन $\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ बराबर है :

(जहाँ c एक समाकलन अचर है)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

Ans. (3)

Sol. $\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{2\sin^2\theta}d\theta$

Let $\sin\theta = t$, $\cos\theta\, d\theta = dt$

$= \int (t^6+t^4+t^2)\sqrt{2t^4+3t^2+6}\,dt = \int (t^5+t^3+t)\sqrt{2t^6+3t^4+6t^2}\,dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\, dt = dz$

$= \frac{1}{12}\int \sqrt{z}\,dz = \frac{1}{18}z^{3/2}+c$

$= \frac{1}{18}[(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta)^{3/2}+C$

$= \frac{1}{18}\left[\left(1-\cos^2\theta\right)\left(2(1-\cos^2\theta)^2+3-3\cos^2\theta+6\right)\right]^{3/2}+C$

$= \frac{1}{18}[(1-\cos^2\theta)(2\cos^4\theta-7\cos^2\theta+11)]^{3/2}+C$

$= \frac{1}{18}[-2\cos^6\theta+9\cos^4\theta-18\cos^2\theta+11]^{3/2}+C$

Topic :- Set Relation

Subtopic:- Hight & Distance

Level :- Easy

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

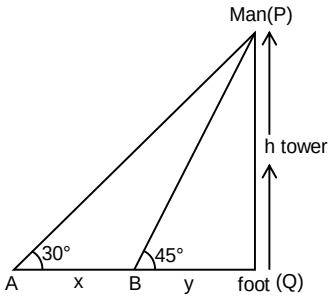
(1) $10(\sqrt{3}-1)$ (2) $10\sqrt{3}$ (3) 10 (4) $10(\sqrt{3}+1)$

एक स्तंभ के शीर्ष से एक पुरुष देख रहा है कि एक निश्चित बिंदु A से एक नाव एक समान गति से स्तंभ की ओर आ रही है। उस समय पुरुष की आँख से नाव का अवनमन कोण 30° है (पुरुष की ऊँचाई पर ध्यान न दें) स्तंभ के आधार (जो पानी की सतह पर है) की तरफ नाव 20 सेकण्ड चलने के पश्चात् एक बिंदु B पर पहुँचती है, जहाँ अवनमन कोण 45°है। नाव के B से स्तंभ के आधार तक पहुँचने में लिया गया समय (सेकण्ड में) है :

(1) $10(\sqrt{3}-1)$ (2) $10\sqrt{3}$ (3) 10 (4) $10(\sqrt{3}+1)$

Ans. (4)

Sol.



$\frac{h}{x+y}=\tan 30°$

$x + y = \sqrt{3}h$ .....(1)

Also

$\frac{h}{y}=\tan 45°$

h = y .....(2)

put in (1)

$x + y = \sqrt{3}y$

$x = (\sqrt{3}-1)y$

$\frac{x}{20}=$'v'speed

$\therefore$time taken to reach

Foot from B

$\Rightarrow\frac{y}{V}$

$\Rightarrow\frac{x}{(\sqrt{3}-1).x}\times 20$

$\Rightarrow 10(\sqrt{3}+1)$

Topic :- S & P (Progression)

Subtopic:- G.P. (M17)

Level :- Easy

5. If $0 <\theta, \phi<\frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta, y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z=\sum_{n=0}^{\infty}\cos^{2n}\theta.\sin^{2n}\phi$ then:

(1)xyz = 4
(2)xy – z = (x + y)z
(3)xy + yz + zx = z
(4)xy + z = (x +y)z

यदि $0 <\theta, \phi<\frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta, y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ तथा $z=\sum_{n=0}^{\infty}\cos^{2n}\theta.\sin^{2n}\phi$ है तो :

(1)xyz = 4
(2)xy – z = (x + y)z
(3)xy + yz + zx = z
(4)xy + z = (x +y)z

Ans. (4)

Sol. $x = 1 + \cos^2\theta + ...\infty$

$x = \frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$ .....(1)

$y = 1 + \sin^2\phi + ...\infty$

$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$ .....(2)

$z = \frac{1}{1-\cos^2\theta.\sin^2\phi} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$

$xz + yz - z = xy$

$xy + z = (x + y)z$

Topic :- 3D

Subtopic:- Equation of Line (M171)

Level :- Easy

6. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

(1) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$ (2) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(3) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$ (4) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

बिंदु (0, 1, 2) से होकर जाने वाली तथा रेखा $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ के लंबवत रेखा का समीकरण है :

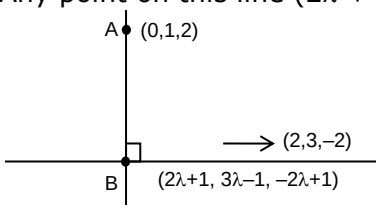
(1) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$ (2) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(3) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$ (4) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

Ans. (1)

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$

Any point on this line $(2\lambda + 1, 3\lambda - 1, -2\lambda+1)$



Direction ratio of given line (2, 3, −2)

Direction ratio of line to be found $(2\lambda + 1, 3\lambda - 2, -2\lambda - 1)$

$\therefore \vec{d}_1.\vec{d}_2 = 0$

$\lambda = 2/17$

Direction ratio of line (21, − 28, −21) ≡ (3, −4, −3) ≡ (−3, 4, 3)

Topic :- Set & Relation

Subtopic:- Mathematical Reasoning

Level :- Easy

7. The statement $A\rightarrow (B \rightarrow A)$ is equivalent to:
(1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \vee B)$ (3) $A \rightarrow (A \rightarrow B)$ (4) $A \rightarrow (A \leftrightarrow B)$
कथन $A\rightarrow (B \rightarrow A)$ निम्न में से किसके तुल्य है ?
(1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \vee B)$ (3) $A \rightarrow (A \rightarrow B)$ (4) $A \rightarrow (A \leftrightarrow B)$

Ans. (2)

Sol. $A \rightarrow (B \rightarrow A)$
$\Rightarrow A \rightarrow (\sim B \vee A)$
$\Rightarrow \sim A \vee (\sim B \vee A)$
$\Rightarrow \sim B \vee (\sim A \vee A)$
$\Rightarrow \sim B \vee t$
$= t$ (tantology)
From options :
(2) $A \rightarrow (A \vee B)$
$\Rightarrow \sim A \vee (A \vee B)$
$\Rightarrow (\sim A \vee A) \vee B$
$\Rightarrow t \vee B$
$\Rightarrow t$

Topic :- Quadratic Equaiton

Subtopic:- Graphical Prob. (M13)

Level :- Easy

8. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is :
(1) 3 (2) 2 (3) 4 (4) 0
पूर्णांक 'k', जिसके लिए असमिका $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$, R में प्रत्येक x के लिए, मान्य है, है
(1) 3 (2) 2 (3) 4 (4) 0

Ans. (1)

Sol. $D < 0$

$$\left(2(3k-1)\right)^2 - 4\left(8k^2 - 7\right) < 0$$

$$4\left(9k^2 - 6k + 1\right) - 4\left(8k^2 - 7\right) < 0$$

$k^2 - 6k + 8 < 0$
$(k-4)(k-2) < 0$
$2 < k < 4$
then $k = 3$

Topic :- Parabola

Subtopic:- Tangent of Parabola (M275)

Level :- Easy

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?
(1) (0, 3) (2) (−6, 0) (3) (4, 5) (4) (5, 4)
परवलय, $y^2 = 6x$ पर एक स्पर्श रेखा खींची गई है जो रेखा $2x + y = 1$ के लंबवत है। तो निम्न में से कौन सा बिंदु इस पर स्थित नहीं है ?
(1) (0, 3) (2) (−6, 0) (3) (4, 5) (4) (5, 4)

Ans. (4)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3 \quad \Rightarrow x - 2y + 6 = 0$

Topic :- Function

Subtopic:- Composite function (M203)

Level :- Medium

10. Let f, g: $N \to N$ such that $f(n + 1) = f(n) + f(1) \ \forall n \in N$ and g be any arbitrary function.Which of the following statements is NOT true ?
(1) f is one-one
(2) If fog is one-one, then g is one-one
(3) If g is onto, then fog is one-one
(4) If f is onto, then $f(n) = n \ \forall n \in N$
माना f, g: $N \to N$ है, जिनके लिए $f(n + 1) = f(n) + f(1) \ \forall n \in N$ है तथा g एक स्वेच्छ फलन हैं। निम्न में से कौन सा कथन सत्य नहीं है ?
(1) f एकैकी है
(2) यदि fog एकैकी है, तो g एकैकी है
(3) यदि g आच्छादक है, तो fog एकैकी है
(4) यदि f आच्छादक है, तो $f(n) = n \ \forall n \in N$ है

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$
$f(2) = 2f(1)$
$f(3) = 3f(1)$
$f(4) = 4f(1)$
....
$f(n) = nf(1)$
f(x) is one-one

Topic :- Complex Number

Subtopic:- Geometry (M265)

Level :- Tough

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is :

(1) $\frac{3}{\sqrt{2}}$ (2) $3\sqrt{2}$ (3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

माना रेखाऐं $(2 - i)z = (2 + i)\bar{z}$ तथा $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (यहाँ $i^2 = -1$) एक वृत C पर अभिलम्ब हैं। यदि रेखा $iz+\bar{z}+1+i=0$, वृत C की स्पर्श रेखा है, तो इसकी त्रिज्या है :

(1) $\frac{3}{\sqrt{2}}$ (2) $3\sqrt{2}$ (3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. (3)

Sol. $(2-i)z=(2+i)\bar{z}$

$\Rightarrow(2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow 2x-ix+2iy+y=2x+ix-2-iy+y$

$\Rightarrow 2ix-4iy=0$

$L_1 : x-2y=0$

$\Rightarrow(2+i)z+(i-2)\bar{z}-4i=0.$

$\Rightarrow(2+i)(x+iy)+(i-2)(x-iy)-4i=0.$

$\Rightarrow 2x+ix+2iy-y+ix-2x+y+2iy-4i=0$

$\Rightarrow 2ix+4iy-4i=0$

$L_2 : x+2y-2=0$

Solve L_1 and L_2 $4y=2$, $y=\frac{1}{2}$

$\therefore x=1$

Centre $\left(1,\frac{1}{2}\right)$

$L_3 : iz+\bar{z}+1+i=0$

$\Rightarrow i(x+iy)+x-iy+1+i=0$

$\Rightarrow ix-y+x-iy+1+i=0$

$\Rightarrow(x-y+1)+i(x-y+1)=0$

Radius = distance from $\left(1,\frac{1}{2}\right)$ to $x-y+1=0$

$r=\frac{1-\frac{1}{2}+1}{\sqrt{2}}$

$r=\frac{3}{2\sqrt{2}}$

Topic :- Trigo Phase - II

Subtopic:- Trigonometric ineqquality (M34)

Level :- Medium

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

(3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

$\theta \in [0, 2\pi]$ के सभी संभव मान, जिनके लिए $\sin 2\theta + \tan 2\theta > 0$ है, निम्न में से किस में है ?

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

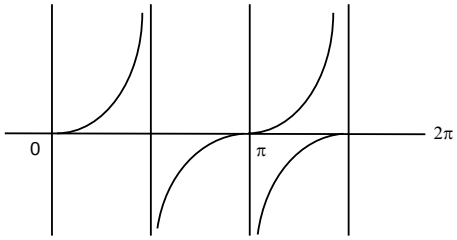
(2) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

(3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

Ans. (2)

Sol.



$\tan 2\theta (1 + \cos 2\theta) > 0$

$2\theta \in \left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

Topic :- St. Line

Subtopic:- Mixed (M91)

Level :- Easy

13. The image of the point (3,5) in the line $x - y + 1 = 0$, lies on :

(1) $(x-2)^2 + (y-4)^2 = 4$ (2) $(x-4)^2 + (y+2)^2 = 16$

(3) $(x-4)^2 + (y-4)^2 = 8$ (4) $(x-2)^2 + (y-2)^2 = 12$

रेखा $x - y + 1 = 0$ में बिंदु (3, 5) का प्रतिबिंब निम्न में से किस पर स्थित है ?

(1) $(x-2)^2 + (y-4)^2 = 4$ (2) $(x-4)^2 + (y+2)^2 = 16$

(3) $(x-4)^2 + (y-4)^2 = 8$ (4) $(x-2)^2 + (y-2)^2 = 12$

Ans. (1)

Sol. Image of P(3, 5) on the line $x - y + 1 = 0$ is

$$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$$

$x = 4, y = 4$

$\therefore$ Image is (4, 4)

Which lies on

$(x-2)^2 + (y-4)^2 = 4$

Topic :- Monotonocity

Subtopic:- Rolle's theorem (M298)

Level :- Medium

14. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (–5, 8) (2) (5, 8) (3) (5, –8) (4) (–5, –8)

यदि $f'\left(\frac{4}{3}\right) = 0$ के साथ फलन $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ के लिए रोले का प्रमेय लागू होता है तो क्रमित युग्म (a, b) बराबर है:

(1) (–5, 8) (2) (5, 8) (3) (5, –8) (4) (–5, –8)

Ans. (2)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$3a - b = 7$ ...(1)

$f'(x) = 3x^2 - 2ax + b$

$$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$$

$\Rightarrow -8a + 3b = -16$ ...(2)

$a = 5, b = 8$

Topic :- Tangent & normal

Subtopic:- Angle Between curves of intersction & ort. (M288)

Level :- Tough

15. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is true ?

(1)a + b = c + d (2)a – b = c – d

(3)ab = $\frac{c+d}{a+b}$ (4)a – c = b + d

यदि वक्र $\frac{x^2}{a}+\frac{y^2}{b}=1$ तथा $\frac{x^2}{c}+\frac{y^2}{d}=1$ एक दूसरे को 90° के कोण पर काटते हैं, तो निम्न में से कौन सा संबंध सत्य है ?

(1)a + b = c + d (2)a – b = c – d

(3)ab = $\frac{c+d}{a+b}$ (4)a – c = b + d

Ans. (2)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ .....(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dx}=\frac{-bx}{ay}$ .....(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ .....(3)

Diff : $\frac{dy}{dx}=\frac{-dx}{cy}$ .....(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$ .....(5)

$(1)-(3) \Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2 + \left(\frac{1}{b}-\frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (using 5)

$\Rightarrow (c - a) - (d - b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

Topic :- Difinite Integration

Subtopic:- Estimation (M114)

Level :- Tough

16. $\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{e}$ (3) 1 (4) 0

$\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ बराबर है :

(1) $\frac{1}{2}$ (2) $\frac{1}{e}$ (3) 1 (4) 0

Ans. (3)

Sol. It is 1^∞ form

$L = e^{\lim\limits_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)}$

$S = 1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+.....+\frac{1}{15}\right)$

$S< 1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right).....+\underbrace{\left(\frac{1}{2^P}+.....+\frac{1}{2^P}\right)}_{2^P \text{ times}}$

$S < 1+ 1 + 1 + 1 +..... +1$

$S < P + 1$

$\therefore L = e^{\lim\limits_{n\to\infty}\frac{(P+1)}{2^P}}$

$\Rightarrow L = e^\circ = 1$

Topic :- P & C

Subtopic:- Distribution of alike object (Beggars Methored) (M218)

Level :- Easy

17. The total number of positive integral solutions (x, y, z) such that xyz = 24 is

(1) 36 (2) 45 (3) 24 (4) 30

xyz = 24 के धन पूर्णांक हलों (x, y, z) की कुल संख्या है :

(1) 36 (2) 45 (3) 24 (4) 30

Ans. (4)

Sol. x.y.z = 24

$x.y.z=2^3.3^1$

Now using beggars method.

3 things to be distributed among 3 persons
Each may receive none, one or more
$\therefore {}^5C_2$ways
Similarly for '1' $\therefore {}^3C_2$ ways
Total ways $= {}^5C_2 \cdot {}^3C_2 = 30$ways

Topic :- Differential Eqaution

Subtopic:- Mixed (M140)

Level :- Medium

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$,then this curve also passes through the point :

(1)(4, 5) (2) (5, 4) (3) (4, 4) (4) (5, 5)

यदि एक वक्र मूलबिंदु से होकर जाता है तथा इसके किसी बिंदु (x, y) पर स्पर्श रेखा की प्रवणता $\frac{x^2-4x+y+8}{x-2}$ है, तो यह वक्र निम्न में से किस बिंदु से भी होकर जाता है ?

(1)(4, 5) (2) (5, 4) (3) (4, 4) (4) (5, 5)

Ans. (4)

Sol. $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{(x-2)}=(x-2)+\frac{y+4}{(x-2)}$

Let $x - 2 = t \Rightarrow dx = dt$
and $y + 4 = u \Rightarrow dy = du$

$\frac{dy}{dx}=\frac{du}{dt}$

$\frac{du}{dt}=t+\frac{u}{t}\Rightarrow\frac{du}{dt}-\frac{u}{t}=t$

I.F $= e^{\int\frac{-1}{t}dt}=e^{-\ln t}=\frac{1}{t}$

$u.\frac{1}{t}=\int t.\frac{1}{t}dt\Rightarrow\frac{u}{t}=t+c$

$\frac{y+4}{x-2} = (x - 2) + c$

Passing through (0, 0)
$c = 0$
$\Rightarrow (y + 4) = (x - 2)^2$

Topic :- D.I.

Subtopic:- Elementray (M114)

Level :- Easy

19. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\leq$ t,is :

(1) $\frac{e+1}{3}$ (2) $\frac{e-1}{3e}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

$\int_{-1}^{1} x^2 e^{[x^3]} dx$, जहाँ [t] महत्तम् पूर्णांक $\leq$ t,is है, का मान है :

(1) $\frac{e+1}{3}$ (2) $\frac{e-1}{3e}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

Ans. (3)

Sol. $I = \int_{-1}^{0} x^2 . e^{-1} dx + \int_{0}^{1} x^2 dx$

$\therefore I = \frac{x^3}{3e}\Big|_{-1}^{0} + \frac{x^3}{3}\Big|_{0}^{1}$

$\Rightarrow I = \frac{1}{3e} + \frac{1}{3}$

Topic :- Probability

Subtopic:- Prob. Of independent & Event (M229)

Level :- Easy

20. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(1) $\frac{1}{8}$ (2) $\frac{1}{27}$ (3) $\frac{3}{4}$ (4) $\frac{3}{8}$

जब एक प्रक्षेपास्त्र किसी जहाज से दागा जाता है, तो इसके अवरुद्ध होने की प्रायिकता $\frac{1}{3}$ है तथा यह दिए होने पर कि यह अवरुद्ध नहीं होता, इसके निशाने पर लगने की प्रायिकता $\frac{3}{4}$ है। यदि जहाज से तीन प्रक्षेपास्त्र स्वतंत्र रूप से दागे जाते हैं, तो सभी तीनों के निशाने पर लगने की प्रायिकता है :

(1) $\frac{1}{8}$ (2) $\frac{1}{27}$ (3) $\frac{3}{4}$ (4) $\frac{3}{8}$

Ans. (1)

Sol. Probabilityof not getting intercepted $= \frac{2}{3}$

Probability of missile hitting target $= \frac{3}{4}$

$\therefore$ Probability that all 3 hit the target $= \left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

Section : Mathematics Section B

Topic :- S & P (Progression)

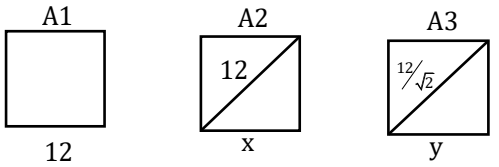
Subtopic:- G.P. (M17)

Level :- Medium

1. Let A_1, A_2, A_3, ... be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is _____.
माना A_1, A_2, A_3, ... वर्ग है जबकि प्रत्येक $n \geq 1$ के लिए, A_n की भुजा की लम्बाई A_{n+1} के विकर्ण की लम्बाई के बराबर है। यदि A_1 की भुजा की लम्बाई 12 cm है, तो n का न्यूनतम मान, जिसके लिए A_n का क्षेत्रफल एक से कम है, है_____।

Ans. (9)

Sol.



$x = \frac{12}{\sqrt{2}}$ $\quad y = \frac{12}{(\sqrt{2})^2}$

$\therefore$ Side lengths are in G.P.

$T_n = \frac{12}{(\sqrt{2})^{n-1}}$

$\therefore$ Area $= \frac{144}{2^{n-1}} < 1 \quad \Rightarrow 2^{n-1} > 144$

Smallest n = 9

Topic :- AUC

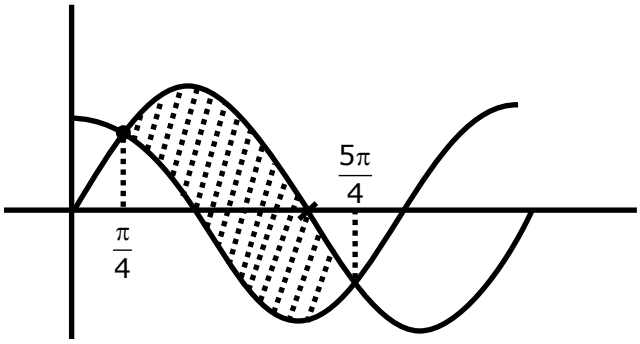
Subtopic:- Bounded between two curve (M143)

Level :- Easy

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _____
साइन तथा कोसाइन फलनों के ग्राफ एक दूसरे को बहुत से बिन्दुओं पर काटते हैं तथा इनके दो क्रमागत प्रतिच्छेदन बिंदुओ के बीच में ये दो ग्राफ एक समान क्षेत्रफल A घेरते है। तो A^4 बराबर है_____।

Ans. (64)

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

Topic :- Hyperbola

Subtopic:- Basic Defⁿ (M273)

Level :- Medium

3. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is _____.

रेखाओं $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ तथा $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ के प्रतिच्छेदन बिंदु का बिंदुपथ एक शांकव है जिसकी उत्केन्द्रता है _____।

Ans. (2)

Sol. $\sqrt{3}kx + ky = 4\sqrt{3}$ ..... (1)

$\sqrt{3}kx - ky = 4\sqrt{3}k^2$ .....(2)

Adding equation (1) & (2)

$2\sqrt{3}kx = 4\sqrt{3}(k^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$ .....(3)

Substracting equation (1) & (2)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$ .....(4)

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$ Hyperbola

$\therefore e^2 = 1 + \frac{48}{16}$

$e = 2$

Topic :- Matrix

Subtopic:- inverse of matrix (M184)

Level :- Medium

4. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _____.

यदि $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ तथा $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ हैं, तो $13(a^2 + b^2)$ बराबर है _____।

Ans. (13)

Sol. $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ $\{\because |I - A| = \sec^2\theta/2\}$

$$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow (I+A)(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$$

$$a = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$b = \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$\therefore a^2 + b^2 = 1$

Topic :- maxima & minima

Subtopic:- Mixed (M308)

Level :- Medium

5. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1 and x = 1. If $\lim_{x\to 0}\frac{f(x)}{x^3} = 1$, then 5.f(2) is equal to _____

माना x में एक बहुपद f(x) की घात 6 है, तथा x^6 का गुणांक एक है और x = –1 तथा x = 1 इसके चरम बिंदु है। यदि $\lim_{x\to 0}\frac{f(x)}{x^3} = 1$ तो 5.f(2) बराबर है _____।

Ans. (144)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & – 1

$\therefore 6 + 5a + 4b + 3 = 0$ & $- 6 + 5a - 4b + 3 = 0$ solving

$$a = -\frac{3}{5} \qquad b = -\frac{3}{2}$$

$$\therefore f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$$

$$\therefore 5.f(2) = 5\left[64 - \frac{96}{5} - 24 + 8\right] = 144$$

Topic :- Differentiability

Subtopic:- Diff. in na interval (M72)

Level :- Medium

6. The number of points, at which the function $f(x) = |2x + 1| - 3|x+2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is _____.
उन बिंदुओ की संख्या, जिन पर फलन $f(x) = |2x + 1| - 3|x+2| + |x^2 + x - 2|$, $x \in R$ अवकलनीय नहीं है, है_____।

Ans. (2)

Sol. $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$$f(x)=\begin{cases} x^2-7 \; ; & x>1 \\ -x^2-2x-3 \; ; & -\frac{1}{2}<x<1 \\ -x^2-6x-5 \; ; & -2<x<\frac{-1}{2} \\ x^2+2x+3; & x<-2 \end{cases}$$

$$\therefore f'(x)=\begin{cases} 2x \; ; & x>1 \\ -2x-3 \; ; & -\frac{1}{2}<x<1 \\ -2x-6; & -2<x<\frac{-1}{2} \\ 2x+2; & x<-2 \end{cases}$$

Check at 1, –2 and $\frac{-1}{2}$

Non. Differentiable at x = 1 and $\frac{-1}{2}$

Topic :- Determinants

Subtopic:- Carmere's Rule (M190)

Level :- Easy

7. If the system of equations
$kx + y + 2z = 1$
$3x - y - 2z = 2$
$-2x - 2y - 4z = 3$
has infinitely many solutions, then k is equal to _____.
यदि समीकरण निकाय
$kx + y + 2z = 1$
$3x - y - 2z = 2$
$-2x - 2y - 4z = 3$
के अनन्त हल हैं, तो k बराबर है_____।

Ans. (21)

Sol. D = 0

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow$k (4 – 4)– 1(–12 –4) +2(–6 –2)

$\Rightarrow$16 – 16 = 0

Also. $D_1 = D_2 = D_3 = 0$

$$\Rightarrow D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow k(-8+6)-1(-12-4)+2(9+4)=0$

$\Rightarrow -2k + 16 + 26 = 0$

$\Rightarrow 2k = 42$

$\Rightarrow k = 21$

Topic :- Vector

Subtopic:- Cross product (M161)

Level :- Medium

8. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is avector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}.\vec{b}=0$, then $\vec{r}.\vec{a}$ is equal to _____

माना तीन सदिश $\vec{a}=\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}$ तथा $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ दिए गए है। यदि $\vec{r}$ एक सदिश है, जिसके लिए $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ तथा $\vec{r}.\vec{b}=0$ है, तो $\vec{r}.\vec{a}$ बराबर है _____।

Ans. (12)

Sol. $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$

$\vec{r}\times\vec{a}-\vec{c}\times\vec{a}=0$

$(\vec{r}-\vec{c})\times\vec{a}=0$

$\therefore \vec{r}-\vec{c}=\lambda\vec{a}$

$\vec{r}=\lambda\vec{a}+\vec{c}$

$\vec{r}.\vec{b}=\lambda\vec{a}.\vec{b}+\vec{c}.\vec{b}=0$

$\Rightarrow \lambda$ (1–2) + 2 = 0

$\Rightarrow \lambda = 2$

$\therefore \vec{r}=2\vec{a}+\vec{c}$

$\vec{r}.\vec{a}=2|\vec{a}|^2+\vec{a}.\vec{c}$

= 2(1 + 4 + 1) + (1 – 2 + 1)

= 12

Topic :- Matrix

Subtopic:- Multiplication of Matrices (M180)

Level :- Medium

9. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _____.

माना $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ है, जहाँ x, y तथा z वास्तविक संख्याऐं हैं, जिनके लिए $x + y + z > 0$ तथा $xyz = 2$ है।

यदि $A^2 = I_3$ है तो $x^3 + y^3 + z^3$ का मान है _____।

Ans. (7)

Sol. $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ $\therefore |A| = \left(x^3+y^3+z^3-3xyz\right)$

$A^2 = I_3$

$|A^2| = 1$

$\therefore (x^3 + y^3 + z^3 - 3xyz)^2 = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$ only as $(x+y+z>0)$

$\Rightarrow x^3 + y^3 + z^3 = 6 + 1 = 7$

Topic :- P & C

Subtopic:- Arrangement of distinet object (M211)

Level :- Medium

10. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is _____.

अंको 1, 2, 3, 4,5 से 100 तथा 1000 के बीच की बनाई जा सकने वाली संख्याओं, यदि कोई भी अंक दोहराया नहीं जाता है तथा संख्याऐं या तो 3 से या 5 से विभाज्य है, की कुल संख्या हैं _____।

Ans. (32)

Sol. □□□ divisible by $\rightarrow 3$

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

□□5 = 12

4×3

Required No. = 24 + 12 – 4 = 32

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at Kota Classroom

- **40 Classes** of each subjects
- **Doubt Clearing** sessions by **Expert faculties**
- **Full Syllabus Tests** to improve your question solving skills
- Thorough learning of concepts with regular classes
- Get **tips & trick** along with sample papers

Course Fee : ₹ **20,000**

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