24th Feb. 2021 | Shift - 1
MATHEMATICS

JEE I NEET I Foundation

MOTION™

25000+
SELECTIONS SINCE 2007

Topic :- Parabola

Subtopic:- Mixed

Level :- Medium

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

1. परवलय, $y^2=4ax$ की नाभि तथा परवलय पर किसी भी बिन्दु को मिलाने वाले रेखाखंडों के मध्य बिन्दुओं का बिन्दु–पथ एक और परवलय है जिसकी नियता है :

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

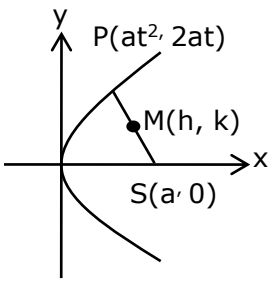
$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$



Topic :- P & C

Subtopic:- Section form diff^{n} objects & propertics

Level :- Medium

2. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 (2) 1050 (3) 1625 (4) 575

2. 6 भारतीयों तथा 8 विदेशियों में से एक वैज्ञानिक कमेटी बनानी हैं, जिसमें कम से कम दो भारतीय हों और उनसे दुगने विदेशी हों; तो ऐसी कमेटी बनाने के तरीकों की संख्या है :

(1) 560 (2) 1050 (3) 1625 (4) 575

Ans. (3)

Sol. (2I, 4F)+ (3I, 6F) + (4I, 8F)

$= {}^{6}C_{2}{}^{8}C_{4} + {}^{6}C_{3}{}^{8}C_{6} + {}^{6}C_{4}\,{}^{8}C_{8}$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

Topic :- 3D

Subtopic:- Mixed

Level :- Easy

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$ (2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$ (4) $6x - 5y + 2z + 10 = 0$

3. उस समतल, जो (1, 2, –3) से होकर जाता है तथा समतलों, $3x + y - 2z = 5$ तथा $2x - 5y - z = 7$ के लम्बवत् है, का समीकरण है:

(1) $3x - 10y - 2z + 11 = 0$ (2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$ (4) $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ 11 (x – 1) + (y – 2) + 17 (z + 3) = 0

$11x + y + 17z + 38 = 0$

Topic :- St. Line

Subtopic:- Mixed

Level :- Easy

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only (2) A only (3) All the three (4) C only

4. एक व्यक्ति एक सरल रेखा पर चल रहा है। इस रेखा द्वारा निर्देशांक अक्षों पर बनाये अंतः खण्डों के व्युत्क्रमों का समान्तर माध्य $\frac{1}{4}$ है। तीन पत्थर A, B तथा C क्रमशः बिन्दुओं (1, 1), (2, 2) तथा (4, 4) पर रखे गये हैं। तो उनमें से कौन–सा/से पत्थर उस व्यक्ति के पथ पर है/हैं ?

(1) B केवल (2) A केवल (3) तीनों (4) C केवल

Ans. (1)

Sol. $\frac{x}{a}+\frac{y}{b} = 1$

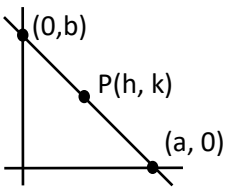
$\frac{h}{a}+\frac{k}{b}=1$ .....(1)

$\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4}$

$\therefore \frac{1}{a}+\frac{1}{b}=\frac{1}{2}$ .....(ii)

$\therefore$ Line passes through fixed point B(2, 2)

(from (1) and (2))



Topic :- Set & Relation

Subtopic:- Mathematical Reasonings

Level :- Easy

5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

5. निम्न में से कौन सा कथन पुनरूक्ति है ?

(1) $A \wedge (A \vee B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Ans. (4)

Sol. $A \wedge (\sim A \vee B) \rightarrow B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim A \vee \sim B \vee B$

$= t$

Topic :- Function

Subtopic:- Classification of fune

Level :- Medium

6. Let $f : R \rightarrow R$ be defined as $f(x) = 2x-1$ and $g: R - \{1\} \rightarrow R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

6. माना $f : R \rightarrow R$ $f(x) = 2x-1$ द्वारा तथा $g: R - \{1\} \rightarrow g(x) = \frac{x - \frac{1}{2}}{x - 1}$ द्वारा परिभाषित है। तो संयुक्त फलन $f(g(x))$:

(1) एकैकी है परन्तु आच्छादक नहीं है (2) आच्छादक है परन्तु एकैकी नहीं है

(3) न एकैकी है और न आच्छादक है (4) एकैकी तथा आच्छादक दोनों है

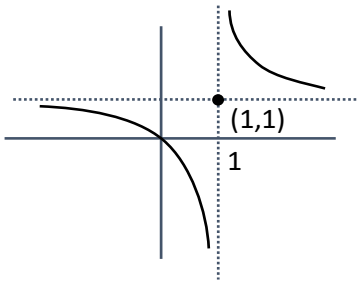
Ans. (4)

Sol. $f(g(x)) = 2g(x) - 1$

$= 2\frac{\left(x - \frac{1}{2}\right)}{x - 1} = \frac{x}{x - 1}$

$f(g(x)) = 1 + \frac{1}{x-1}$

one-one, into



Topic :- Continuity

Subtopic:- Mcontinuity at a point

Level :- Medium

7. If $f:R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 1

(2) discontinuous at all integral values of x except at x = 1

(3) continuous only at x = 1

(4) continuous for every real x

7. एक फलन $f:R \to R$, $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$ द्वारा परिभाषित है, जबकि [.] महत्तम पूर्णांक फलन है, तो f :

(1) केवल x = 1 पर असंतत् है

(2) x के सभी पूर्णांक मानों x = 1 के अतिरिक्त, पर असंतत है

(3) केवल x = 1 पर संतत है

(4) प्रत्येक वास्तविक x के लिए संतत् है

Ans. (4)

Sol. Doubtful points are $x = n, n \in I$

$L.H.L = \lim\limits_{x \to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$

$R.H.L = \lim\limits_{x \to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$

$f(n) = 0$

Hence continuous.

Topic :- Monotonocity

Subtopic:- Checking monotonocity at a point or inari interval

Level :- Easy

8. The function $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$:

(1) increases in $\left[\frac{1}{2}, \infty\right)$ (2) decreases $\left(-\infty, \frac{1}{2}\right]$

(3) increases in $\left(-\infty, \frac{1}{2}\right]$ (4) decreases $\left[\frac{1}{2}, \infty\right)$

8. फलन $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$:

(1) $\left[\frac{1}{2}, \infty\right)$ में वर्धमान है (2) $\left(-\infty, \frac{1}{2}\right]$ में ह्रासमान है

(3) $\left(-\infty, \frac{1}{2}\right]$ में वर्धमान है (4) $\left[\frac{1}{2}, \infty\right)$ में ह्रासमान है

Ans. (1)

Sol. $f'(x) = (2x - 1)(x - \sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2}, \infty\right)$

and $f'(x) \leq 0$ in $x \in \left(-\infty, \frac{1}{2}\right]$

Topic :- 3D

Subtopic:- Line and Plane

Level :- Easy

9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is:

(1) $\sqrt{38}$ (2) $19\sqrt{2}$ (3) $2\sqrt{19}$ (4) 38

9. रेखा $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ तथा समतल x + y + z = 17 के प्रतिच्छेदन बिन्दु की बिन्दु (1, 1, 9) से दूरी है:

(1) $\sqrt{38}$ (2) $19\sqrt{2}$ (3) $2\sqrt{19}$ (4) 38

Ans. (1)

Sol. $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$

$\Rightarrow x = \lambda+3,\ y = 2\lambda+4,\ z = 2\lambda+5$

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda = \frac{5}{5} = 1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

Topic :- Definite integration

Subtopic:- newton's labintz (M113)

Level :- Easy

10. $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) 0 (3) $\frac{1}{15}$ (4) $\frac{3}{2}$

10. $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$ बराबर है :

(1) $\frac{2}{3}$ (2) 0 (3) $\frac{1}{15}$ (4) $\frac{3}{2}$

Ans. (1)

Sol. $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \sin\sqrt{t}dt}{x^3} = \lim\limits_{x\to 0} \frac{\left(\sin|x|\right)2x}{3x^2} = \lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)\times\frac{2}{3} = \frac{2}{3}$

Topic :- Set & Relation

Subtopic:- Height & Distance

Level :- Easy

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

11. दो ऊर्ध्वाधर पोल, 150 m की दूरी पर हैं तथा उन में से एक की ऊँचाई दूसरे की ऊँचाई से तीन गुना है। यदि एक दर्शक, जो पोलों के पादों को मिलाने वाली रेखा के मध्यबिन्दु पर है, पोलों के शीर्षों के उन्नयन कोण पूरक पाता है, तो छोटे पोल की ऊँचाई (मीटरों में) है:

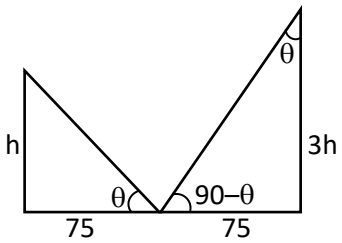
(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

Ans. (4)

Sol. $\tan\theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}m$



Topic :- Tangent & Normal

Subtopic:- Tangent & Normal Cutting Curve again (M285)

Level :- Medium

12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

12. यदि वक्र $y = x^3$ के बिन्दु $P(t, t^3)$ पर खींची गई स्पर्श रेखा वक्र को फिर से बिन्दु Q पर मिलती है, तो उस बिन्दु की कोटि जो रेखा–खण्ड PQ को आंतरिक अनुपात 1 : 2 में काटता है, है :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

Ans. (1)

Sol. Equation of tangent at $P(t, t^3)$

$(y - t^3) = 3t^2(x - t)$ .....(1)

Now solve the above equation with

$y = x^3$ .....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\frac{2t^3 + (-8t^3)}{3} = -2t^3$

Topic :- AUc

Subtopic:- Area between Two Curve (M143)

Level :- Medium

13. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi + 3\sqrt{3}$ (2) $12\pi + 3\sqrt{3}$ (3) $12\pi - 3\sqrt{3}$ (4) $24\pi - 3\sqrt{3}$

13. वृत्त, $x^2+y^2=36$ के उस भाग का क्षेत्रफल (वर्ग इकाइयों में), जो परवलय $y^2=9x$ के बाहर है, है:

(1) $24\pi + 3\sqrt{3}$ (2) $12\pi + 3\sqrt{3}$ (3) $12\pi - 3\sqrt{3}$ (4) $24\pi - 3\sqrt{3}$

Ans. (4)

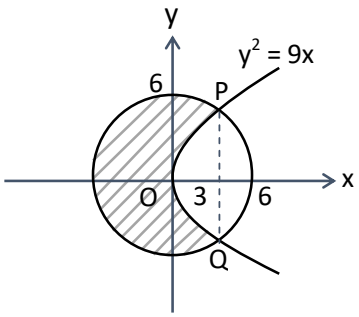
Sol. The curves intersect at point $(3, \pm 3\sqrt{3})$

Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9 - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$



Topic :- Indefinite Integration

Subtopic:- Integration Between Substitution (M121)

Level :- Easy

14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

14. यदि $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$ है, जबकि c एक समाचलन अचर है, तो क्रमित युग्म (a, b) बराबर है:

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

Ans. (2)

Sol. put sin x + cos x = t $\Rightarrow$ 1 + sin 2x = t^2

$\Rightarrow$ (cos x – sin x) dx = dt

$$\therefore I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C$$

$\Rightarrow$ a = 1 and b = 3

Topic :- Differential Equation

Subtopic:- Variable separable

Level :- Easy

15. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

15. किसी प्रजाति की समय 't' पर जनसंख्या, P = P(t) अवकल समीकरण, $\frac{dP}{dt}$ = 0.5P – 450 को संतुश्ट करती है। यदि P(0) = 850 है, तो वह समय, जब प्रजाति की जनसंख्या शून्य हो जाती है, है :

(2) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

Ans. (2)

Sol. $\frac{dp}{dt} = \frac{p-900}{2}$

$$\int_{850}^{0} \frac{dp}{p-900} = \int_{0}^{t} \frac{dt}{2}$$

$$\ell n|P-900|\Big|_{850}^{0} = \frac{t}{2}$$

$$\ell n|900| - \ell n|50| = \frac{t}{2}$$

$$\frac{t}{2} = \ell n|18|$$

$$\Rightarrow t = 2\ell n18$$

Topic :- Binomial Theorem

Subtopic:- Collection of Binomial Coeff.

Level :- Medium

16. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ...+ {}^{14}C_{11}$ is:

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

16. $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ...+ {}^{14}C_{11}$ का मान है :

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

Ans. (4)

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - ..... - 15\ {}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r .r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$$

$= 15\ (-{}^{14}C_0 + {}^{14}C_1 - .....-{}^{14}C_{14}) = 15\ (0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$= S_1 + S_2 = 2^{13} - 14$

Topic :- Parabola

Subtopic:- Baye's Theoram (M231)

Level :- Medium

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

17. एक सामान्य पासा कुछ बार उछाला जाता है। यदि दोबार विषम संख्या आने की प्रायिकता, तीन बार समसंख्या आने की प्रायिकता के बराबर है, तो एक विषम संख्या के विषम बार आने की प्रायिकता है:

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

Ans. (2)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

Topic :- Q.E

Subtopic:- Sum of roots & Product of roots (M8)

Level :- Easy

18. Let p and q be two positive number such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

18. यदि p तथा q दो धनात्मक संख्याएँ है, जिनके लिए $p + q = 2$ तथा $p^4 + q^4 = 272$ हैं, तो p तथा q जिस समीकरण के मूल हैं, वह है :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 + 16pq + 2p^2 q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Now

$x^2 - (p + q)x + pq = 0$

$x^2 - 2x + 16 = 0$

Topic :- Trigo -1

Subtopic:- Trigonometric Series

Level :- Easy

19. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

19. यदि $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty\right)\log_e 2}$ समीकरण $t^2 - 9t + 8 = 0$, को संतुष्ट करता है, तो $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ का मान है :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

Ans. (3)

Sol. $e^{\left(\cos^2 x+\cos^4 x+\ldots\ldots\infty\right)\ell n2} = 2^{\cos^2 x+\cos^4 x+\ldots\ldots\infty}$

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x}=\frac{2}{1+\sqrt{3}\cot x}=\frac{2}{4}=\frac{1}{2}$$

Topic :- Determinant

Subtopic:- Csamme's Rule

Level :- Medium

20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

20. रैखिक समीकरण निकाय

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

असंगत है, यदि :

(1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

Ans. (4)

Sol. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$

$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$

$80 - 12 + 20m - 36 - 60m \neq 0$

$40m \neq 32 \Rightarrow m \neq \frac{4}{5}$

$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$

$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$

$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$

$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$

$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \frac{4}{5}$

Section – B

Topic :- Matric

Subtopic:- Adjoint of Matric (M183)

Level :- Tough

1. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____

1. माना $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$ है, जबकि $\alpha \in R$ है। माना $Q = [q_{ij}]$ एक आव्यूह है, जिसके लिए $PQ = kI_3$, किसी शून्येतर $k \in R$ के लिए, है। यदि $q_{23} = -\frac{k}{8}$ तथा $|Q| = \frac{k^2}{2}$, हैं, तो $\alpha^2 + k^2$ बराबर है _____।

Ans. 17

Sol. As $PQ = KI \Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(adjP)I \Rightarrow Q = \frac{k}{(20+12\alpha)} \begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)}(-3\alpha - 4) = \frac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \quad \Rightarrow \quad \alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|} \quad \Rightarrow \quad \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \quad \Rightarrow k = 4$

Topic :- Probability

Subtopic:- Matually exclus, each, indepe (M229)

Level :- Tough

2. Let $B_i(i=1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _____

2. माना एक प्रतिदर्श समष्टि में $B_i(i=1, 2, 3)$ तीन स्वतंत्र घटनाएं है। केवल B_1 के होने की प्रायिकता α है, केवल B_2 के होने की प्रायिकता β है तथा केवल B_3 के होने की प्रायिकता γ है। माना किसी भी घटना B_i के न होनें की प्रायिकता p है, तथा ये चारों प्रायिकताएं समीकरणों $(\alpha-2\beta)p=\alpha\beta$ तथा $(\beta-3\gamma)p=2\beta\gamma$ को संतुष्ट करती हैं। (सभी प्रायिकताएं अन्तराल (0, 1) में हैं)। तो $\frac{P(B_1)}{P(B_3)}$ बराबर है _____ ।

Ans. 6

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha$

$\Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma$

$\Rightarrow (1-x)(1-y)(1-z) = p$

$(\alpha-2\beta)p = \alpha\beta$

$(x(1-y)(1-z)-2y(1-x)(1-z))(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$x - xy - 2y + 2xy = xy$

$x = 2y \quad \ldots(1)$

Similarly $(\beta-3r)p = 2\beta r$

$\Rightarrow y = 3z \quad \ldots(2)$

From (1) & (2)

x = 6z

Now

$\frac{x}{z} = 6$

Topic :- Trigo Phase - 1

Subtopic:- Trigonometric graph equaiton (M33)

Level :- Medium

3. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____

3. α का न्यूनतम मान, जिसके लिए समीकरण $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ का अन्तराल $\left(0, \frac{\pi}{2}\right)$ में कम से कम एक हल है, है _____।

Ans. 9

Sol. $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

Let sinx = t $\quad \because x \in \left(0, \frac{\pi}{2}\right) \Rightarrow 0 < t < 1$

$f(t) = \frac{4}{t} + \frac{1}{1-t}$

$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$

$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$

$= \frac{(t - 2(1-t))(t + 2(1-t))}{t^2(1-t)^2}$

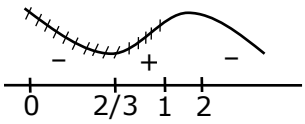
$= \frac{(3t - 2)(2 - t)}{t^2(1-t)^2}$

f_{min} at $t = \frac{2}{3}$

$$\alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

$= 6 + 3$

$= 9$



Topic :- Cricle

Subtopic:- Basic defⁿ of cricle (M92)

Level :- Easy

4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2,1), then its radius is _____

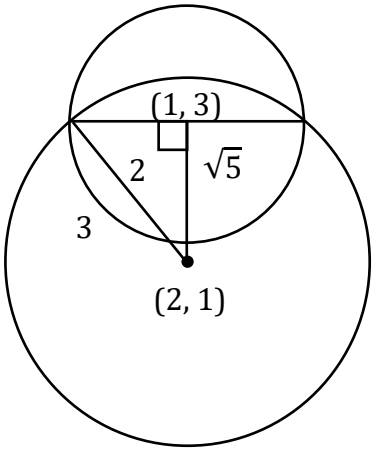
4. वृत्त, $x^2 + y^2 - 2x - 6y + 6 = 0$ का कोई एक व्यास, किसी और वृत्त 'C' की एक जीवा है। यदि वृत्त 'C' का केन्द्र (2,1) 'C' है, तो इसकी त्रिज्या बराबर है _____।

Ans. 3

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$



Topic :- ITF

Subtopic:- Series (M131)

Level :- Medium

5. $\lim\limits_{x\to\infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to _____

5. $\lim\limits_{x\to\infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ बराबर है _____।

Ans. 1

Sol. $\tan\left(\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim\limits_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

Topic :- Definite Integration

Subtopic:- elementary (M108)

Level :- Medium

6. If $\int\limits_{-a}^{a}(|x|+|x-2|)dx = 22$,(a > 2) and [x] denotes the greatest integer $\le$ x, then $\int\limits_{a}^{-a}(x+[x])dx$ is equal to _____

6. यदि $\int\limits_{-a}^{a}(|x|+|x-2|)dx = 22$,(a > 2) है तथा [x], महत्तम पूर्णांक $\le$ x को दर्शाता है, तो $\int\limits_{a}^{-a}(x+[x])dx$ बराबर है _____।

Ans. 3

Sol. $\int\limits_{-a}^{0}(-2x+2)dx+\int\limits_{0}^{2}(x+2-x)dx+\int\limits_{2}^{a}(2x-2)dx = 22$

$x^2-2x\,|_0^{-a}+2x\,|_0^2+x^2-2x\,|_2^a = 22$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int\limits_{3}^{-3}(x+[x])dx = -\left(\int\limits_{-3}^{3}(x+[x])dx\right) = -\left(\int\limits_{-3}^{3}[x]dx\right)$

$= -(-3-2-1+0+1+2) = 3$

Topic :- Vector

Subtopic:- Coplaner & non Coplanar (M159)

Level :- Medium

7. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}\cdot\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _____

7. माना तीन सदिश $\vec{a},\vec{b}$ तथा $\vec{c}$ इस प्रकार हैं कि $\vec{c}$ सदिशों $\vec{a}$ था $\vec{b}$ के समतल में है, $\vec{a}.\vec{c}=7$ है तथा $\vec{b}$, सदिश $\vec{c}$ के लम्बवत है, जबकि $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ तथा $\vec{b}=2\hat{i}+\hat{k}$, हैं, तो $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ बराबर है _____ ।

Ans. **75**

Sol. $\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$=\lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7\Rightarrow\ 3\lambda+5\lambda+6\lambda=7$

$\lambda=\frac{1}{2}$

$\therefore\ 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$=\ 2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$

Topic :- Set & Relation

Subtopic:- Sets

Level :- Tough

8. Let A = {n∈N : n is a 3-digit number}

B = {9k + 2 : k∈N}

and C : {9k + ℓ : k∈N} for someℓ (0 < ℓ < 9)

If the sum of all the elements of the set A ∩ (B ∪ C) is 274×400, then ℓ is equal to __

8. माना A = {n∈N : n एक 3- अंकों की संख्या है}

B = {9k + 2 : k∈N}

तथा C : {9k + ℓ : k∈N} किसी ℓ (0 < ℓ < 9) के लिए

यदि समुच्चय A ∩ (B ∪ C) के सभी अवयवों का योग 274×400 है, तो ℓ बराबर है __ ।

Ans. **5**

Sol. 3 digit number of the form 9K + 2 are {101, 109, ...992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093) = s_1 = 54650

$274 \times 400 = s_1 + s_2$

$274 \times 400 = \frac{100}{2}[101 + 992] + s_2$

$274 \times 400 = 50 \times 1093 + s_2$

$s_2 = 109600 - 54650$

$s_2 = 54950$

$s_2 = 54950 = \frac{100}{2}\left[(99+\ell)+(990+\ell)\right]$

$1099 = 2\ell + 1089$

$\ell = 5$

Topic :- Complex Number

Subtopic:- Mixed (M271)

Level :- Medium

9. If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ $\left(z \in C \text{ and } i=\sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___

9. यदि α के न्यूनतम तथा अधिकतम वास्तविक मान, जिनके लिए समीकरण $z+\alpha|z-1|+2i=0$ $\left(z \in C \text{ and } i=\sqrt{-1}\right)$ का हल है, क्रमशः p तथा q हैं, तो $4(p^2 + q^2)$ बराबर है ____।

Ans. 10

Sol. $x + iy + \alpha\sqrt{(x-1)^2+y^2} + 2i = 0$

$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \frac{x^2}{x^2-2x+5} \Rightarrow x^2(\alpha^2-1)-2x\alpha^2+5\alpha^2 = 0$

$x \in R \Rightarrow D \geq 0$

$4\alpha^4 - 4(\alpha^2-1)5\alpha^2 \geq 0$

$\alpha^2\left[4\alpha^2 - 2\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[-16\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[\alpha^2-\frac{5}{4}\right]\le 0$

$0\le\alpha^2\le\frac{5}{4}$

$\therefore \alpha^2\in\left[0,\frac{5}{4}\right]$

$\therefore \alpha\in\left[-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}\right]$

then $4[(q)^2+(p)^2] = 4\left[\frac{5}{4}+\frac{5}{4}\right] = 10$

Topic :- Matrix

Subtopic:- Mutiplication of Matrics (M180)

Level :- Tough

10. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is __

10. माना M कोई 3×3 आव्यूह है जिसके अवयव समुच्च {0, 1, 2} से लिये गए हैं। इस तरह के आव्यूहों की अधिकतम संख्या, जिसके लिए M^TM के विकर्ण के अवयवों का योग 7 है, है __ ।

Ans. 540

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

${}^9C_2 = 36$

Case II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

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