



**JEE
MAIN
MARCH
2021**

**18th March 2021 | Shift - 2
MATHEMATICS**

JEE | NEET | Foundation

MOTION™

25000+
SELECTIONS SINCE 2007

SECTION -A

Determinant

1. Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$$

Has a non-trivial solution. Then which of the following is true?

- (1) $\mu = 6, \lambda \in \mathbb{R}$ (2) $\lambda = 2, \mu \in \mathbb{R}$ (3) $\lambda = 3, \mu \in \mathbb{R}$ (4) $\mu = -6, \lambda \in \mathbb{R}$

1. माना ऐसिक समीकरण निकाय

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$$

का एक अनुच्छ हल है तो निम्न में से कौन सा सत्य है?

- (1) $\mu = 6, \lambda \in \mathbb{R}$ (2) $\lambda = 2, \mu \in \mathbb{R}$ (3) $\lambda = 3, \mu \in \mathbb{R}$ (4) $\mu = -6, \lambda \in \mathbb{R}$

Ans. (1)

Sol. For non trivial solution

$$\Delta = 0$$

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$4(-3 - 2) - \lambda(6 - \mu) + 2(4 + \mu) = -20 - 6\lambda + \lambda\mu + 8 + 2\mu$$

$$= 12 - 6\lambda + \lambda\mu + 2\mu$$

$$\Rightarrow -12 - 6\lambda + (\lambda + 2)\mu$$

$$\mu = 6, \lambda \in \mathbb{R}$$

Complex Number

2. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of $\triangle ABC$ is 2, then the height of the pole is equal to:

- (1) $\frac{1}{\sqrt{3}}$ (2) $\sqrt{3}$ (3) $2\sqrt{3}$ (4) $\frac{2\sqrt{3}}{3}$

2. एक त्रिकोणीय पार्क ABC के अंदर एक पोल उर्ध्वाधर खड़ा है। माना पार्क के प्रत्येक कोने से पोल के शीर्ष का उन्नयन कोण $\frac{\pi}{3}$ है। यदि $\triangle ABC$ के परिवृत्त की त्रिज्या 2 है, तो पोल की ऊँचाई है:

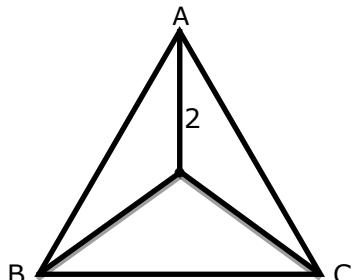
- (1) $\frac{1}{\sqrt{3}}$ (2) $\sqrt{3}$ (3) $2\sqrt{3}$ (4) $\frac{2\sqrt{3}}{3}$

Ans. (3)

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Sol.



$$\tan 60^\circ = \frac{h}{2} \Rightarrow h = 2\sqrt{3}$$

Central tendency & differention

Ans. (4)

Sol. Given series

(a,a,a.....n times), (-a, -a, -a,..... n times)

$$\text{Now } \bar{x} = \frac{\sum x_i}{2n} = 0$$

as $x_i \rightarrow x_i + b$

then $\bar{x} \rightarrow \bar{x} + b$

$$\text{So, } \bar{x} + b = 5 \Rightarrow b = 5$$

No change in S.D. due to change in origin

$$\sigma = \sqrt{\frac{\sum x_i^2}{2n} - (\bar{x})^2} = \sqrt{\frac{2na^2}{2n} - 0}$$

$$20 = \sqrt{a^2} \Rightarrow a = 20$$

$$a^2 + b^2 = 425$$

Definite Integration

4. Let $g(x) = \int_0^x f(t) dt$, where f is continuous function in $[0, 3]$ such that $\frac{1}{3} \leq f(t) \leq 1$ for all $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which $g(3)$ lies is:

(1) $[1, 3]$ (2) $\left[-1, -\frac{1}{2}\right]$ (3) $\left[-\frac{3}{2}, -1\right]$ (4) $\left[\frac{1}{3}, 2\right]$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Ans. (4)

$$\text{Sol. } \int_0^1 \frac{1}{3} dt + \int_{-1}^3 0 \cdot dt \leq g(3) \leq \int_0^1 1 \cdot dt + \int_{-1}^3 \frac{1}{2} dt$$

$$\frac{1}{3} \leq g(3) \leq 2$$

Trigonometry Phase-I

5. If $15\sin^4 \alpha + 10\cos^4 \alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6 \alpha + 8\csc^6 \alpha$ is equal to:
(1) 250 (2) 500 (3) 400 (4) 350

5. यदि किसी $\alpha \in \mathbb{R}$ के लिए $15\sin^4 \alpha + 10\cos^4 \alpha = 6$ है, तो $27\sec^6 \alpha + 8\csc^6 \alpha$ का मान बराबर है:
(1) 250 (2) 500 (3) 400 (4) 350

Ans. (1)

Sol. $15 \sin^4\theta + 10 \cos^4\theta = 6$

$$\Rightarrow 15 \sin^4 \theta + 10(1 - \sin^2 \theta)^2 = 6$$

$$\Rightarrow 25 \sin^4 \theta - 20 \sin^2 \theta + 4 = 0$$

$$\Rightarrow (5 \sin^2 \theta - 2)^2 = 0 \Rightarrow \sin^2 \theta = \frac{2}{5}, \cos^2 \theta = \frac{3}{5}$$

$$\text{Now } 27 \csc^6 \theta + 8 \sec^6 \theta = 27 \left(\frac{125}{27} \right) + 8 \left(\frac{125}{8} \right) = 250$$

Function

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\begin{aligned} &\Rightarrow \frac{3x - 2}{x - 1} + \frac{x + 3}{2} = \frac{13}{2} \\ &\Rightarrow 2(3x - 2) + (x - 1)(x + 3) = 13(x - 1) \\ &\Rightarrow x^2 - 5x + 6 = 0 \\ &\Rightarrow x = 2 \text{ or } 3 \end{aligned}$$

Progressions

Ans (1)

$$\text{Sol. } S_{4n} - S_{2n} = 1000$$

$$\Rightarrow \frac{4n}{2} (2a + (4n-1)d) - \frac{2n}{2} (2a + (2n-1)d) = 1000$$

$$\Rightarrow 2an + 6n^2d - nd = 1000$$

$$\Rightarrow \frac{6n}{2} (2a + (6n-1)d) = 3000$$

$$\therefore S_{6n} = 3000$$

Circle

- 8.** Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:

(1) $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$ (2) $\left(2, \pm \frac{3}{2}\right)$ (3) $(1, \pm 2)$ (4) $(0, \pm \sqrt{3})$

8. माना $S_1 : x^2 + y^2 = 9$ तथा $S_2 : (x - 2)^2 + y^2 = 1$ हैं तो एक चर वृत्त S, जो S_1 को अंदर से स्पर्श करता है तथा S_2 को बाहर से स्पर्श करता है, के केन्द्र का बिन्दुपथ हमेंशा निम्न में से किन बिन्दुओं से होकर जाता है ?

(1) $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$ (2) $\left(2, \pm \frac{3}{2}\right)$ (3) $(1, \pm 2)$ (4) $(0, \pm \sqrt{3})$

Ans. (2)

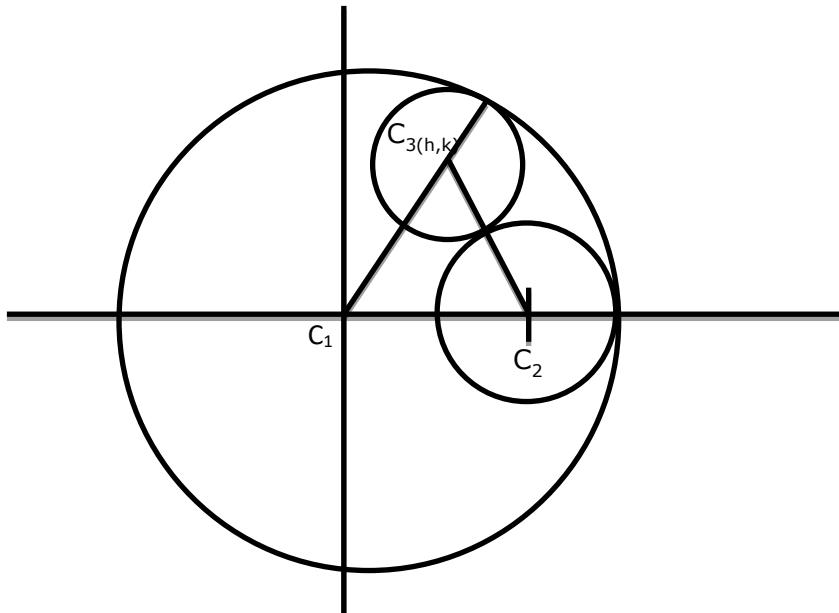
Sol. $C_1 : (0,0)$, $r_1 = 3$

$$C_3 : (2, 0), r_3 \equiv 1$$

Let centre of variable circle be $C_3(h, k)$ and radius be r .

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in



$$C_3C_1 = 3 - r$$

$$C_2C_3 = 1 + r$$

$$C_3C_1 + C_2C_3 = 4$$

So locus is ellipse whose foci are C_1 & C_2

And major axis is $2a = 4$ and $2ae = C_1C_2 = 2$

$$\Rightarrow e = \frac{1}{2}$$

$$\Rightarrow b^2 = 4 \left(1 - \frac{1}{4}\right) = 3$$

Centre of ellipse is midpoint of C_1 & C_2 is $(1, 0)$

$$\text{Equation of ellipse is } \frac{(x-1)^2}{2^2} + \frac{y^2}{(\sqrt{3})^2} = 1$$

Now by cross checking the option $\left(2, \pm \frac{3}{2}\right)$ satisfied it.

Solution of Triangle

9. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of $\triangle ABC$, then $(R+r)$ is equal to

(1) $2\sqrt{2}$

(2) $3\sqrt{2}$

(3) $7\sqrt{2}$

(4) $\frac{9}{\sqrt{2}}$

9. माना एक समबाहु त्रिभुज ABC का केन्द्रक मूलबिन्दु पर है। माना इस त्रिभुज की एक भुजा सरल रेखा $x + y = 3$ के अनुदिश है। यदि $\triangle ABC$ के परिवृत्त तथा अंतवृत्त की त्रिज्याएँ क्रमशः R तथा r हैं, तो $(R+r)$ बराबर है:

(1) $2\sqrt{2}$

(2) $3\sqrt{2}$

(3) $7\sqrt{2}$

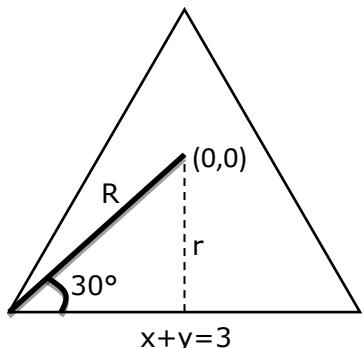
(4) $\frac{9}{\sqrt{2}}$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Ans. (4)

Sol.



$$r = \frac{|0 + 0 - 3|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\sin 30^\circ = \frac{r}{R} = \frac{1}{2}$$

$$R = 2r$$

$$\text{So, } r + R = 3r = 3 \times \left(\frac{3}{\sqrt{2}} \right) = \frac{9}{\sqrt{2}}$$

3-D

10. In a triangle ABC, if $|\vec{BC}| = 8$, $|\vec{CA}| = 7$, $|\vec{AB}| = 10$, then the projection of the vector \vec{AB} on \vec{AC} is equal to:

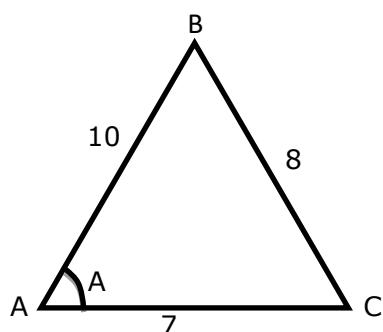
- (1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

10. एक त्रिभुज ABC में यदि $|\vec{BC}| = 8$, $|\vec{CA}| = 7$, $|\vec{AB}| = 10$ है, तो सदिश \vec{AB} का सदिश \vec{AC} पर प्रक्षेप बराबर है:

- (1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Ans. (2)

Sol.



Projection of AB on AC is = $AB \cos A$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$= 10 \cos A$$

By cosine rule

$$\cos A = \frac{10^2 + 7^2 - 8^2}{2 \cdot 10 \cdot 7}$$

$$= \frac{85}{140}$$

$$\Rightarrow 10 \cos A = 10 \left(\frac{85}{140} \right) = \frac{85}{14}$$

Probability

11. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to:

(1) $\frac{80}{243}$ (2) $\frac{32}{625}$ (3) $\frac{128}{625}$ (4) $\frac{40}{243}$

11. माना 5 स्वतंत्र परीक्षणों के एक द्विपद बंटन में ठीक एक और दो सफलताओं की प्रायिकता क्रमशः 0.4096 तथा 0.2048 है तो ठीक तीन सफलताओं की प्रायिकता है:

(1) $\frac{80}{243}$ (2) $\frac{32}{625}$ (3) $\frac{128}{625}$ (4) $\frac{40}{243}$

Ans. (2)

Sol. ${}^5C_1 p^1 q^4 = 0.4096 \dots (1)$

$${}^5C_2 p^2 q^3 = 0.2048 \dots (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{q}{2p} = 2 \Rightarrow q = 4p$$

$$p + q = 1 \Rightarrow P = \frac{1}{5}, q = \frac{4}{5}$$

$$P(\text{exactly 3}) = {}^5C_3 (p)^3 (q)^2 = {}^5C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^2$$

$$= 10 \times \frac{1}{125} \times \frac{16}{25} = \frac{32}{625}$$

VECTOR

12. Let \vec{a} and \vec{b} be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}) \right)$ and \vec{a} is equal to :

(1) $\sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$ (2) $\cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$ (3) $\sin^{-1} \left(\frac{1}{\sqrt{6}} \right)$ (4) $\cos^{-1} \left(\frac{1}{\sqrt{2}} \right)$

12. माना \vec{a} तथा \vec{b} दो शून्येतर सदिश हैं जो एक दूसरे के लम्बवत् हैं तथा $|\vec{a}| = |\vec{b}|$ है। यदि $|\vec{a} \times \vec{b}| = |\vec{a}|$ है, तो सदिशों $\left(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}) \right)$ तथा \vec{a} के बीच का कोण बराबर है:

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(4) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Ans. (2)

Sol. Given $|\vec{a} \times \vec{b}| = |\vec{a}| = |\vec{b}|$

$$\cos \theta = \frac{\vec{a} \left(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}) \right)}{|\vec{a}| \cdot |\vec{a} + \vec{b} + \vec{a} \times \vec{b}|}$$

Let $|\vec{a}| = a$

$$\cos \theta = \frac{a^2 + 0 + 0}{a \times \sqrt{a^2 + a^2 + a^2}} = \frac{a^2}{a^2 \sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Complex Number

13. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to:

(1) $\frac{1}{2}$

(2) 4

(3) 2

(4) $\frac{1}{4}$

13. माना एक समिश्र संख्या $w = 1 - \sqrt{3}i$ है। माना एक अन्य समिश्र संख्या z इस प्रकार है कि $|zw| = 1$ तथा $\arg(z) - \arg(w) = \frac{\pi}{2}$ है तो मूलबिन्दु z तथा w शीर्षों के त्रिभुज का क्षेत्रफल है:

(1) $\frac{1}{2}$

(2) 4

(3) 2

(4) $\frac{1}{4}$

Ans. (1)

Sol. $w = 1 - \sqrt{3}i$

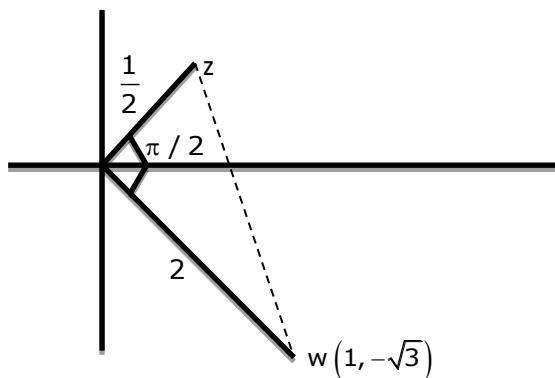
$$|w| = 2$$

$$|zw| = 1 \Rightarrow |z| = \frac{1}{|w|} = \frac{1}{2}$$

$$\arg(z) - \arg(w) = \pi / 2$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in



$$\text{Area of } \Delta = \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = \frac{1}{2}$$

Area Under the Curve

14. The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to:

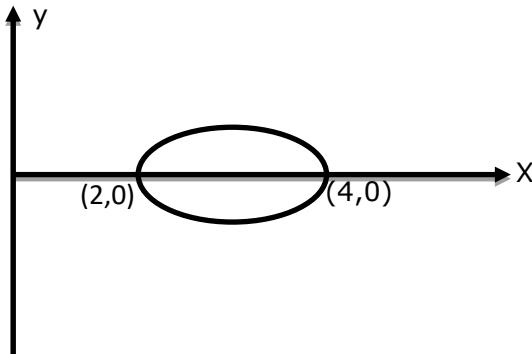
(1) $\frac{3\pi}{2}$ (2) $\frac{\pi}{16}$ (3) $\frac{\pi}{8}$ (4) $\frac{3\pi}{8}$

14. वक्र $4y^2 = x^2(4-x)(x-2)$ द्वारा परिबद्ध का क्षेत्रफल है:

(1) $\frac{3\pi}{2}$ (2) $\frac{\pi}{16}$ (3) $\frac{\pi}{8}$ (4) $\frac{3\pi}{8}$

Ans. (1)

Sol. domain of $4y^2 = x^2(4-x)(x-2)$



$$\text{Area of loop} = 2 \times \frac{1}{2} \times \int_{2}^{4} x \sqrt{(4-x)(x-2)} dx$$

$$\begin{aligned} \text{Put } & x = 4 \sin^2 \theta + 2 \cos^2 \theta \\ & dx = (8 \sin \theta \cos \theta - 4 \cos \theta \sin \theta) d\theta \\ & = 4 \sin \theta \cos \theta d\theta \\ & = \int_0^{\pi/2} (4 \sin^2 \theta + 2 \cos^2 \theta) \sqrt{(2 \cos^2 \theta)(2 \sin^2 \theta)} (4 \sin \theta \cos \theta) d\theta \end{aligned}$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\begin{aligned}
 &= \int_0^{\pi/2} (4 \sin^2 \theta + 2 \cos^2 \theta) 8 (\cos \theta \sin \theta)^2 \\
 &= \int_0^{\pi/2} 32 \sin^4 \theta \cos^2 \theta d\theta + \int_0^{\pi/2} 16 \sin^2 \theta \cos^4 \theta d\theta
 \end{aligned}$$

Using wallistheorm

$$\begin{aligned}
 &= 32 \cdot \frac{3 \cdot 1 \cdot 1}{6 \cdot 4 \cdot 2} \frac{\pi}{2} + 16 \cdot \frac{3 \cdot 1 \cdot 1}{6 \cdot 4 \cdot 2} \frac{\pi}{2} \\
 &= \pi + \pi / 2 = 3\pi / 2
 \end{aligned}$$

MATRIX

15. Define a relation R over a class of $n \times n$ real matrices A and B as “ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ”

The which of the following is true?

- (1) R is reflexive, symmetric but not transitive
- (2) R is symmetric, transitive but not reflexive,
- (3) R is an equivalence relation
- (4) R is reflexive, transitive but not symmetric

15. $n \times n$ के वास्तविक आव्यूहों A तथा B के एक समूह पर एक संबंध R निम्न प्रकार से परिभाषित है: “ARB यदि और केवल यदि एक व्युत्क्रमणीय आव्यूह P का अस्तित्व है जिसके लिए $PAP^{-1} = B$ है”।

तो निम्न में से कौन-सा सत्य है ?

- (1) R स्वतुल्य और सममित है परन्तु संक्रामक नहीं है
- (2) R सममित और संक्रामक है परन्तु स्वतुल्य नहीं है
- (3) R एक तुल्यता संबंध है
- (4) R स्वतुल्य और संक्रामक है परन्तु सममित नहीं है

Ans. (3)

Sol. For reflexive

$$(B, B) \in R \Rightarrow B = PBP^{-1}$$

Which is true for $P = I$

$\therefore R$ is Reflexive

For symmetry

As $(B, A) \in R$ for matrix P

$$B = PAP^{-1} \Rightarrow P^{-1}B = P^{-1}PAP^{-1}$$

$$\Rightarrow P^{-1}BP = IAP^{-1}P = IAI$$

$$P^{-1}BP = A \Rightarrow A = P^{-1}BP$$

$\therefore (A, B) \in R$ for matrix P^{-1}

$\therefore R$ is symmetric

For transitivity

$$B = PAP^{-1} \text{ and } A = PCP^{-1}$$

$$\Rightarrow B = P(PCP^{-1})P^{-1}$$

$$\Rightarrow B = P^2C(P^{-1})^2 \Rightarrow B = P^2C(P^2)^{-1}$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$\therefore (B, C) \in R$ for matrix P^2

$\therefore R$ is transitive

So R is equivalence

Mathematics reasoning

16. If P and Q are two statements, then which of the following compound statement is a tautology?

- (1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$ (2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
 (3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$ (4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

16. यदि P तथा Q दो कथन हैं, तो निम्न में से कौनसा मिश्र कथन पुनरुत्पत्ति है ?

- (1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$ (2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
 (3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$ (4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

Ans. (2)

Sol.
$$\begin{aligned} (P \Rightarrow Q) \wedge \sim Q \\ \equiv (\sim P \vee Q) \wedge \sim Q \\ \equiv (\sim P \vee \sim Q) \vee (Q \wedge \sim Q) \\ \equiv \sim (P \vee Q) \end{aligned}$$

Now,

- (1) $\sim (P \vee Q) \Rightarrow P$
 $\equiv (P \vee Q) \vee P$
 $\equiv P \vee Q$
- (2) $\sim (P \vee Q) \Rightarrow \sim P$
 $\equiv (P \vee Q) \vee \sim P$
 $\equiv T$
- (3) $\sim (P \vee Q) \Rightarrow (P \wedge Q)$
 $\equiv (P \vee Q) \vee (P \wedge Q)$
 $\equiv P \vee Q$
- (4) $\sim (P \vee Q) \Rightarrow Q$
 $\equiv (P \vee Q) \vee Q$
 $\equiv P \vee Q$

Parabola, Ellipse & Hyperbola

17. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x -axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P , then area of ΔQFR is equal to:

- (1) $\sqrt{6} - 1$ (2) $4\sqrt{6} - 1$ (3) $4\sqrt{6}$ (4) $\frac{7}{\sqrt{6}} - 2$

Toll Free : 1800-212-1799

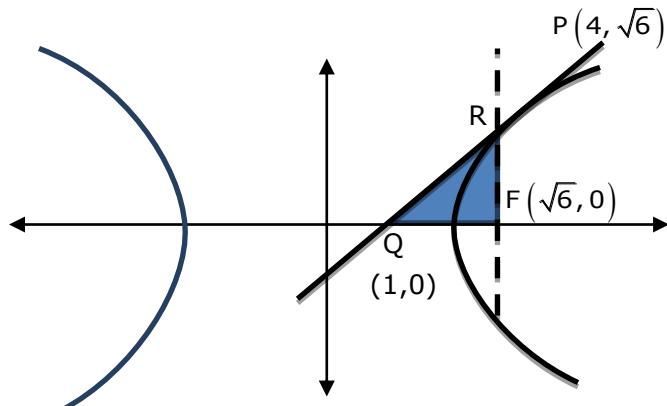
www.motion.ac.in | Email : info@motion.ac.in

17. एक अतिपरवलय $H : x^2 - 2y^2 = 4$ का विचार कीजिए। माना बिंदु $P(4, \sqrt{6})$ पर स्पर्श रेखा x-अक्ष को Q पर मिलती है तथा नाभि जीवा को R(x_1, y_1), $x_1 > 0$ पर मिलती है। यदि H की नाभि F बिंदु P के निकट है, तो ΔQFR का क्षेत्रफल बराबर है—

- (1) $\sqrt{6} - 1$ (2) $4\sqrt{6} - 1$ (3) $4\sqrt{6}$ (4) $\frac{7}{\sqrt{6}} - 2$

Ans. (4)

Sol.



Tangent at $P(4, \sqrt{6})$

$$4(x) - 2 \cdot \sqrt{6}(y) = 4$$

$$\Rightarrow 2x - \sqrt{6}y = 2 \dots (1)$$

For Q, put $y = 0$

$Q(1,0)$

Equation of Latus rectum:

$$x = ae = 2\sqrt{\frac{3}{2}} = \sqrt{6} \quad \dots (2)$$

Solving (1) & (2), we get

$$R\left(\sqrt{6}, 2 - \frac{2}{\sqrt{6}}\right)$$

$$\text{Area of } \Delta QFR = \frac{1}{2} \times QF \times FR$$

$$= \frac{1}{2} \left(\sqrt{6} - 1\right) \left(2 - \frac{2}{\sqrt{6}}\right)$$

$$= \frac{7}{\sqrt{6}} - 2$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Continuity

18. Let $f : R \rightarrow R$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & \text{If } x < 0 \\ b, & \text{If } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{If } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal

- (1) -2 (2) $-\frac{5}{2}$ (3) $-\frac{3}{2}$ (4) -3

18. माना फलन $f : R \rightarrow R$ निम्न द्वारा परिभाषित है:

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & \text{If } x < 0 \\ b, & \text{If } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{If } x > 0 \end{cases}$$

यदि $x = 0$ पर f संतत है तो $a + b$ का मान बराबर है:

- (1) -2 (2) $-\frac{5}{2}$ (3) $-\frac{3}{2}$ (4) -3

Ans. (3)

Sol. 'f' is continuous at $x = 0$

$$\Rightarrow f(0^-) = f(0) = f(0^+)$$

$$f(0^-) = \lim_{x \rightarrow 0^-} \frac{\sin(a+1)x + \sin 2x}{2x}$$

$$= \lim_{x \rightarrow 0^-} \left\{ \frac{\sin(a+1)x}{(a+1)x} \cdot \frac{(a+1)}{2} + \frac{\sin(2x)}{2x} \right\}$$

$$= \frac{a+1}{2} + 1 \quad \dots(1)$$

$$f(0^+) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x \rightarrow 0^+} \frac{bx^3}{b \cdot x^{5/2} \cdot (\sqrt{x+bx^3} + \sqrt{x})}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{1+bx^2} + 1}$$

$$= \frac{1}{2} \quad \dots(2)$$

$$f(0) = b \quad \dots(3)$$

From (1), (2) and (3)

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\therefore \frac{a+1}{2} + 1 = \frac{1}{2} = b$$

$$\Rightarrow a = -2 \text{ & } b = \frac{1}{2}$$

$$\text{Thus, } a + b = -3/2$$

Differential equation

19. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)((y+1)e^{x/2} - x)$, $0 < x < 2.1$, with

$y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to:

$$(1) \frac{e^{5/2}}{(1+e^2)^2}$$

$$(2) \frac{5e^{1/2}}{(e^2+1)^2}$$

$$(3) -\frac{2e^2}{(1+e^2)^2}$$

$$(4) \frac{-e^{3/2}}{(e^2+1)^2}$$

19. माना अवकल समीकरण $\frac{dy}{dx} = (y+1)((y+1)e^{x/2} - x)$, $0 < x < 2.1$, $y(2) = 0$ का हल $y = y(x)$ है तो $x = 1$ पर $\frac{dy}{dx}$ का मान बराबर है:

$$(1) \frac{e^{5/2}}{(1+e^2)^2}$$

$$(2) \frac{5e^{1/2}}{(e^2+1)^2}$$

$$(3) -\frac{2e^2}{(1+e^2)^2}$$

$$(4) \frac{-e^{3/2}}{(e^2+1)^2}$$

Ans. (4)

Sol. $\frac{dy}{dx} = (y+1)\left((y+1)e^{\frac{x^2}{2}} - x\right)$

$$\Rightarrow \frac{-1}{(y+1)^2} \frac{dy}{dx} - x\left(\frac{1}{y+1}\right) = -e^{\frac{x^2}{2}}$$

$$\text{Put, } \frac{1}{y+1} = z$$

$$-\frac{1}{(y+1)^2} \cdot \frac{dy}{dx} = \frac{dz}{dx}$$

$$\therefore \frac{dz}{dx} + z(-x) = -e^{\frac{x^2}{2}}$$

$$I.F = e^{\int -x dx} = e^{-\frac{x^2}{2}}$$

$$z \left(e^{-\frac{x^2}{2}} \right) = - \int e^{-\frac{x^2}{2}} \cdot e^{\frac{x^2}{2}} dx = - \int 1 dx = -x + C$$

$$\Rightarrow \frac{e^{-\frac{x^2}{2}}}{y+1} = -x + C \dots (1)$$

Given $y = 0$ at $x = 2$

Put in (1)

$$\frac{e^{-2}}{0+1} = -2 + C$$

$$C = e^{-2} + 2 \dots (2)$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

From (1) and (2)

$$y + 1 = \frac{e^{-x^2/2}}{e^{-2} + 2 - x}$$

Again, at $x = 1$

$$\Rightarrow y + 1 = \frac{e^{\frac{y}{2}}}{e^{\frac{y}{2}} + 1}$$

$$\Rightarrow y + 1 = \frac{e^{\frac{3}{2}}}{e^2 + 1}$$

$$\therefore \left. \frac{dy}{dx} \right|_{x=1} = \frac{e^{\frac{3}{2}}}{e^2 + 1} \left(\frac{e^{\frac{3}{2}}}{e^2 + 1} \times e^{\frac{1}{2}} - 1 \right)$$

$$= - \frac{e^{\frac{3}{2}}}{(e^2 + 1)^2}$$

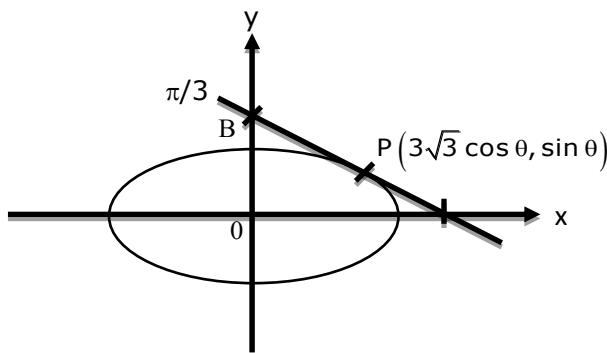
Tangents Normals

- 20.** Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3} \cos \theta, \sin \theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

- 20.** माना दीर्घवृत्त $\frac{x^2}{27} + y^2 = 1$ के बिन्दु $(3\sqrt{3} \cos \theta, \sin \theta)$, $\theta \in \left(0, \frac{\pi}{2}\right)$ पर एक स्पर्श रेखा खींची गई है तो θ का वह मान, जिसके लिए इस स्पर्श रेखा द्वारा अक्षों पर बनाए गए अंतःखण्डों का योगफल निम्नतम है, बराबर है:

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (2) Sol.



Equation of tangent

$$\frac{x}{3\sqrt{3}} \cos \theta + y \sin \theta = 1$$

$$A\left(\frac{3\sqrt{3}}{\cos \theta}, 0\right), B\left(0, \frac{1}{\sin \theta}\right)$$

$$\text{Now sum of intercept} = \frac{3\sqrt{3}}{\cos \theta} + \frac{1}{\sin \theta}$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\text{Let } y = 3\sqrt{3} \sec \theta + \cos \sec \theta$$

$$y' = 3\sqrt{3} \sec \theta \tan \theta - \cos \sec \theta \cot \theta$$

$$y' = 0 \Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

SECTION -B

3-D

1. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to _____.
1. माना P एक समतल है जिसमें रेखा $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ स्थित है तथा जो रेखा $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$ के समांतर है। यदि बिन्दु $(1, -1, \alpha)$ समतल P पर है, तो $|5\alpha|$ का मान बराबर है _____.

Ans. (12)

Sol. DR's of normal $\vec{n} \equiv \vec{b}_1 \times \vec{b}_2$

$$\vec{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{i} \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}$$

$$(34, -13, -25)$$

$$P \equiv 34(x-1) - 13(y+6) - 25(z+5) = 0$$

$Q(1, -1, \alpha)$ lies on P.

$$\Rightarrow 34(1-1) - 13(-1+6) - 25(\alpha+5) = 0$$

$$\Rightarrow -25(\alpha+5) = 65$$

$$\Rightarrow +5\alpha = -38$$

$$\Rightarrow |5\alpha| = 38$$

Binomial Theorem

2. If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$. Then the value of α is equal to _____.

2. यदि $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$ है तो α का मान बराबर है _____.

Ans. (160)

Sol. $T_r = r!((r+1)(r+2)(r+3) - 9r - 1)$

$$= (r+3)! - 9r.r! - r!$$

$$= (r+3)! - 9(r+1-1)r! - r!$$

$$= (r+3)! - 9(r+1)! + 8r!$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$= \{(r+3)! - (r+1)!\} - 8 \{(r+1)! - r!\}$$

$$\text{Now, } \sum_{r=1}^{10} T_r = \{13! + 12! - 3! - 2!\} - 8 \{11! - 1!\}$$

$$= 13! + 12! - 8(11!)$$

$$= (13 \times 12 + 12 - 8)11!$$

$$= 160 \times 11!$$

Thus, $\alpha = 160$

Binomial Theorem

3. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right]^{10}$, $x \neq 1$, is equal to _____.

3. $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right]^{10}$, $x \neq 1$, के प्रसार में x से स्वतंत्र पद बराबर है _____.

Ans. (210)

Sol. Given, $\left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10} = (x^{1/3} - x^{-1/2})^{10}$

$$\text{General term, } T_{r+1} = {}^{10} C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$$

For term independent of x

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow r = 4$$

$$\text{Therefore required term, } T_5 = {}^{10} C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Binomial Theorem

4. Let ${}^n C_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$.

$$\text{If } \sum_{k=0}^{10} (2^2 + 3k) {}^n C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}, \alpha, \beta \in \mathbb{R}, \text{ then } \alpha + \beta \text{ is equal to _____.}$$

4. माना $(1+x)^n$ के प्रसार में x^r का द्विपद गुणांक ${}^n C_r$ है। यदि $\sum_{k=0}^{10} (2^2 + 3k) {}^n C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}, \alpha, \beta \in \mathbb{R}$ है तो $\alpha + \beta$ बराबर है _____.

Bonus

Sol. n must be equal to 10

$$\sum_{k=0}^{10} (2^2 + 3k) {}^n C_k$$

$$= \sum_{k=0}^{10} (4 + 3k) {}^n C_k$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$= 4 \sum_{k=0}^{10} {}^n C_k + 3 \sum_{k=0}^{10} k^n C_k$$

$$= 4(2^{10}) + 3 \times 10 \times 2^9$$

$$= 19 \times 2^{10}$$

$$\therefore \alpha = 0 \text{ and } \beta = 19$$

Thus, $\alpha + \beta = 19$

Definite Integration

5. Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^1 P(x)dx = 18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to _____.
5. माना घात 3 का एक वास्तविक बहुपद $P(x)$ है, जो $x = -3$ पर शून्य हो जाता है। माना $P(x)$ का स्थानीय निम्नतम $x = 1$ पर, स्थानीय अधिकतम $x = -1$ तथा $\int_{-1}^1 P(x)dx = 18$ है तो बहुपद $P(x)$ के सभी गुणांकों का योगफल बराबर है _____.

Ans. (8)

Sol. $P'(x) = a(x+1)(x-1)$

$$\therefore P(x) = \frac{ax^3}{3} - ax + C$$

$$P(-3) = 0 \text{ (given)}$$

$$\Rightarrow a(-9+3) + C = 0$$

$$\Rightarrow 6a = C \quad \dots(i)$$

$$\text{Also, } \int_{-1}^1 P(x)dx = 18 \Rightarrow \int_{-1}^1 \left(a\left(\frac{x^3}{3} - x\right) + C \right) dx = 18$$

$$\Rightarrow 0 + 2C = 18 \Rightarrow C = 9$$

from(i)

$$a = \frac{3}{2}$$

$$\therefore P(x) = \frac{x^3}{2} - \frac{3}{2}x + 9$$

$$\text{Sum of co-efficient} = -1 + 9 = 8$$

3-D

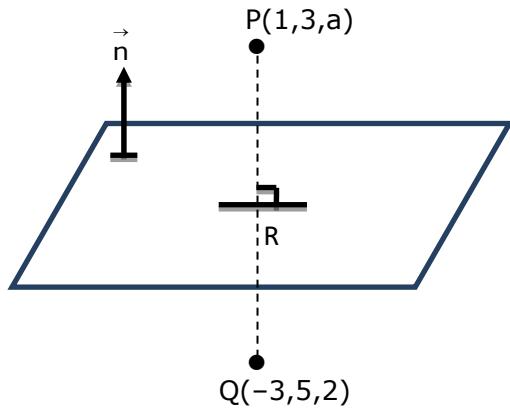
6. Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be $(-3, 5, 2)$. Then, the value of $|a+b|$ is equal to _____.
6. माना समतल $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ के सापेक्ष बिन्दु $(1, 3, a)$ का दर्पण प्रतिबिम्ब $(-3, 5, 2)$ है तो $|a+b|$ का मान बराबर है _____.

Ans. (1)

Sol.

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in



$$\text{Plane : } 2x - y + z = b$$

$$R \equiv \left(-1, 4, \frac{a+2}{2} \right) \rightarrow \text{on plane}$$

$$\therefore -2 - 4 + \frac{a+2}{2} = b$$

$$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10 \quad \dots(1)$$

$PQ <4, -2, a-2>$

$$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2} \Rightarrow a-2 = 2 \Rightarrow a = 4, b = -3$$

$$\therefore |a+b| = 1$$

Function

7. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + x g(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to _____.
 7. यदि $f(x)$ तथा $g(x)$ दो बहुपद हैं जिनके लिए बहुपद $P(x) = f(x^3) + x g(x^3)$, $x^2 + x + 1$ से विभाज्य है, तो $P(1)$ बराबर है _____.

Ans. 0

Sol. roots of $x^2 + x + 1$ are ω and ω^2 now

$$Q(\omega) = f(1) + \omega g(1) = 0 \dots(1)$$

$$Q(\omega^2) = f(1) + \omega^2 g(1) = 0 \dots(2)$$

Adding (1) and (2)

$$\Rightarrow 2f(1) - g(1) = 0$$

$$\Rightarrow g(1) = 2f(1)$$

$$\Rightarrow f(1) = g(1) = 0$$

Therefore, $Q(1) = f(1) + g(1) = 0 + 0 = 0$

MATRIX

8. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in \mathbb{N}$ for which $P^n = 5I - 8P$ is equal to _____.
 8. यदि I एक 2×2 क्रम की इडेंटिटी मैट्रिक्स है और $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. तो $P^n = 5I - 8P$ के लिए $n \in \mathbb{N}$ का मान ज्ञात करें।

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

8. माना I, कोटि 2×2 का तत्समक आव्यूह है तथा $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ है तो $n \in \mathbb{N}$ का वह मान, जिसके लिए $P^n = 5I - 8P$ है, बराबर है _____.

Ans. (6)

$$P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}$$

$$P^6 = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$\text{and } 5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 8 \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$\Rightarrow P^6 = 5I - 8P$$

Thus, $n = 6$

Limits

9. Let $f : R \rightarrow R$ satisfy the equation $f(x + y) = f(x). f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for any $x \in R$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1)$ is equal to _____.
9. माना $f : R \rightarrow R$ समीकरण $f(x + y) = f(x). f(y) \forall x, y \in R$ को संतुष्ट करता है तथा किसी भी $x \in R$ के लिए $f(x) \neq 0$ है। यदि फलन f बिन्दु $x = 0$ पर अवकलनीय है तथा $f'(0) = 3$ है, तो $\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1)$ बराबर है _____.

Ans. (3)

Sol. $f(x + y) = f(x) . f(y)$ then

$$\Rightarrow f(x) = a^x$$

$$\Rightarrow f'(x) = a^x \ell n a$$

$$\Rightarrow f'(0) = \ell n a = 3 \text{ (given } f'(0) = 3\text{)}$$

$$\Rightarrow a = e^3$$

$$\therefore f(x) = (e^3)^x = e^{3x}$$

$$\text{Now, } \lim_{h \rightarrow 0} \frac{f(h) - 1}{h} = \lim_{h \rightarrow 0} \left(\frac{e^{3h} - 1}{3h} \times 3 \right) = 1 \times 3 = 3$$

Area Under the Curve

10. Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{(x^2 - y^2)} dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^\pi$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + b$, then the value of $10(\alpha + \beta)$ is equal to _____.
10. माना अवकल समीकरण $xdy - ydx = \sqrt{(x^2 - y^2)} dx$, $x \geq 1$ का हल $y = y(x)$ है तथा $y(1) = 0$ है। यदि रेखाओं $x = 1$, $x = e^\pi$, $y = 0$ तथा $y = y(x)$ द्वारा घिरे क्षेत्र का क्षेत्रफल $\alpha e^{2\pi} + b$ है, तो $10(\alpha + \beta)$ का मान बराबर है _____.

Ans. (4)

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Sol. $xdy - ydx = \sqrt{x^2 - y^2} dx \Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x} \sqrt{1 - \frac{y^2}{x^2}} dx \Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n|x| + c$$

At $x = 1, y = 0 \Rightarrow c = 0$

$$y = x \sin(\ell n x)$$

$$A = \int_1^{e^\pi} x \sin(\ell n x) dx$$

$$x = e^t, dx = e^t dt = \int_0^\pi e^{2t} \sin(t) dt$$

$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5} (2 \sin t - \cos t) \right)_0^\pi = \frac{1 + e^{2\pi}}{5}$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5}$$

$$\text{Thus, } 10(\alpha + \beta) = 4$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in