



**JEE
MAIN
MARCH
2021**

**18th March 2021 | Shift - 1
MATHEMATICS**

JEE I NEET I Foundation

MotionTM

25000+
SELECTIONS SINCE 2007

SECTION – A

Function

1. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions : $f + g$, $f - g$, f/g , g/f , $g - f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 < x \leq 1$ (2) $0 \leq x < 1$ (3) $0 \leq x \leq 1$ (4) $0 < x < 1$

यदि फलन $f(x) = \sqrt{x}$ तथा $g(x) = \sqrt{1-x}$ हैं, तो फलनों $f + g$, $f - g$, f/g , g/f , $g - f$ जहाँ $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$ हैं, का समान (common) प्रांत है :

(1) $0 < x \leq 1$ (2) $0 \leq x < 1$ (3) $0 \leq x \leq 1$ (4) $0 < x < 1$

Ans. (4)

$$f + g = \sqrt{x} + \sqrt{1-x}$$

$$\Rightarrow x \geq 0 \text{ & } 1-x \geq 0 \Rightarrow x \in [0,1]$$

$$f - g = \sqrt{x} - \sqrt{1-x}$$

$$\Rightarrow x \geq 0 \text{ & } 1-x \geq 0 \Rightarrow x \in [0,1]$$

$$f/g = \frac{\sqrt{x}}{\sqrt{1-x}}$$

$$\Rightarrow x \geq 0 \text{ & } 1-x > 0 \Rightarrow x \in [0,1)$$

$$g/f = \frac{\sqrt{1-x}}{\sqrt{x}}$$

$$\Rightarrow 1-x \geq 0 \text{ & } x > 0 \Rightarrow x \in (0,1]$$

$$g - f = \sqrt{1-x} - \sqrt{x}$$

$$\Rightarrow 1-x \geq 0 \text{ & } x \geq 0 \Rightarrow x \in [0,1]$$

$$\Rightarrow x \in (0,1)$$

DETERMINANT

2. Let α, β, γ be the roots of the equations, $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in \mathbb{R}$ and a, b and $a, b \neq 0$). If the system of the equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$; $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solutions, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 1 (3) 0 (4) 3

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

माना α, β, γ समीकरण $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in \mathbb{R}$ तथा $a, b \neq 0$) के वास्तविक मूल हैं। यदि u, v, w में समीकरण निकाय $\alpha u + \beta v + \gamma w = 0; \beta u + \gamma v + \alpha w = 0; \gamma u + \alpha v + \beta w = 0$ का अतुच्छ हल है, तो $\frac{a^2}{b}$ का मान है :

Ans. (4)

Sol. $x^3 + ax^2 + bx + c = 0$

For non-trivial solutions,

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0$$

$$\alpha + \beta + \gamma \left[\alpha + \beta + \alpha^2 - 3 \sum \alpha \beta \right] = 0$$

$$(-a) [a^2 - 3b] = 0$$

$$a^2 = 3b \quad \because a \neq 0 \quad \Rightarrow \quad \frac{a^2}{b} = 3$$

Complex Number

3. If the equation $a|z|^2 + \overline{az} + \alpha z + d = 0$ represents a circle where a, d are real constants, then which of the following condition is correct?

- (1) $|\alpha|^2 - ad \neq 0$ (2) $|\alpha|^2 - ad > 0$ and $a \in \mathbb{R} - \{0\}$
 (3) $\alpha = 0, a, d \in \mathbb{R}^+$ (4) $|\alpha|^2 - ad \geq 0$ and $a \in \mathbb{R}$

यदि समीकरण $a|z|^2 + \bar{\alpha}z + \alpha\bar{z} + d = 0$ एक वृत्त को निरूपित करता है, जहाँ a, d वास्तविक अचर हैं, तो निम्न में से कौन सा सत्य है ?

Ans. (2)

Sol. $a|z|^2 + \bar{\alpha}z + \bar{\alpha}\bar{z} + d = 0$

$$zz' + \left(\frac{\alpha}{a}\right)z' + \left(\frac{-\alpha}{a}\right)z + \frac{d}{a} = 0 \quad \text{Centre} = -\frac{\alpha}{a}$$

$$r = \sqrt{\left|\frac{\alpha}{a}\right|^2 - \frac{d}{a}} \Rightarrow \left|\frac{\alpha}{a}\right|^2 \geq \frac{d}{a} \Rightarrow |\alpha|^2 \geq ad$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Progressions

4. $\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \dots + \frac{1}{(201)^2 - 1}$ is equal to:

- (1) $\frac{101}{404}$ (2) $\frac{101}{408}$ (3) $\frac{99}{400}$ (4) $\frac{25}{101}$

$\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \dots + \frac{1}{(201)^2 - 1}$ बराबर है :

- (1) $\frac{101}{404}$ (2) $\frac{101}{408}$ (3) $\frac{99}{400}$ (4) $\frac{25}{101}$

Ans. (4)

$$\text{Sol. } S = \sum_{r=1}^{100} \frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \frac{1}{(2r+2) \cdot 2(r)}$$

$$\therefore S = \frac{1}{4} \sum_{r=1}^{100} \left[\frac{1}{r} - \frac{1}{r+1} \right]$$

$$S = \frac{1}{4} \left(\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{100} - \frac{1}{101} \right) \right)$$

$$\therefore S = \frac{1}{4} \left[\frac{100}{101} \right] = \frac{25}{101}$$

Straight Line

5. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:

- (1) 3 (2) 2 (3) 1 (4) 0

m के पूर्णांक मानों, जिनके लिए रेखाओं $3x + 4y = 9$ तथा $y = mx + 1$ के प्रतिच्छेदन बिंदु का भुज भी एक पूर्णांक है, की संख्या है :

- (1) 3 (2) 2 (3) 1 (4) 0

Ans. (2)

$$\text{Sol. } 3x + 4(mx + 1) = 9$$

$$x(3 + 4m) = 5$$

$$x = \frac{5}{(3 + 4m)}$$

$$(3 + 4m) = \pm 1, \pm 5$$

$$4m = -3 \pm 1, -3 \pm 5$$

$$4m = -4, -2, -8, 2$$

$$m = -1, -\frac{1}{2}, -2, \frac{1}{2}$$

Two integral value of m

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

DETERMINANT

6. The solutions of the equation $\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, are:

(1) $\frac{\pi}{6}, \frac{5\pi}{6}$ (2) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{\pi}{12}, \frac{\pi}{6}$

समीकरण $\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, के हल हैं :

(1) $\frac{\pi}{6}, \frac{5\pi}{6}$ (2) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{\pi}{12}, \frac{\pi}{6}$

Ans. (2)

Sol. $R_1 \rightarrow R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 + \cos^2 x & \cos^2 x \\ 0 & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

$$\therefore 2 + 8 \sin 2x - 4 \sin 2x = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2} \quad \Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

Differentiability

7. If $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

(1) $\frac{5}{2}, -\frac{3}{2}$ (2) $-\frac{1}{2}, \frac{3}{2}$ (3) $\frac{1}{2}, \frac{1}{2}$ (4) $\frac{1}{2}, -\frac{3}{2}$

यदि $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$

अपने प्रांत के प्रत्येक बिंदु पर अवकलनीय है, तो a तथा b के मान क्रमशः हैं :

(1) $\frac{5}{2}, -\frac{3}{2}$ (2) $-\frac{1}{2}, \frac{3}{2}$ (3) $\frac{1}{2}, \frac{1}{2}$ (4) $\frac{1}{2}, -\frac{3}{2}$

Ans. (2)

Sol. $f(x)$ is continuous at $x = 1 \Rightarrow 1 = a + b$

$f(x)$ is differentiable at $x = 1 \Rightarrow -1 = 2a$

$$\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

VECTOR

- 8.** A vector \vec{a} has components $3p$ and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If with respect to new system, \vec{a} has components $p+1$ and $\sqrt{10}$, then a value of p is equal to:

एक आयाताकार कार्तीय प्रणाली के सापेक्ष एक सदिश \vec{a} के घटक $3p$ तथा 1 हैं। इस प्रणाली को मूलबिंदु के सापेक्ष वामावृत्त दिशा में किसी कोण तक घुमाया जाता है। यदि नई प्रणाली के सापेक्ष \vec{a} के घटक $p+1$ तथा $\sqrt{10}$ हैं, तो p का एक मान बराबर है :

Ans. (2)

Sol. $\left| \vec{a} \right|_{\text{old}} = \left| \vec{a} \right|_{\text{new}}$

$$(3p)^2 + 1 = (P+1)^2 + 10$$

$$9p^2 - p^2 - 2p - 10 = 0$$

$$8p^2 - 2p - 10 = 0$$

$$4p^2 - p - 5 = 0$$

$$4p^2 - 5p + 4p - 5 = 0$$

$$(4p - 5)(p + 1) = 0$$

$$p = \frac{5}{4}, -1$$

Permutation Combination

9. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664 (2) 122664 (3) 122234 (4) 22264

अकों 1, 2, 2 तथा 3 से बनाई जा सकने वाली सभी 4 अंको की भिन्न संख्याओं का योगफल है :

(1) 26664 (2) 122664 (3) 122234 (4) 22264

Ans. (1)

Sol.	1	2	2	3
	1	2	3	2
	1	3	2	2
	3	1	2	2
	3	2	1	2
	3	2	2	1
	2	1	3	2
	2	3	1	2
	2	2	1	3
	2	2	3	1
	2	3	2	1
	2	1	2	3
	2	6	6	4

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Progressions

12. The value of $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots}}}}$ is equal to:

$$3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots}}}}$$

- (1) $3 + 2\sqrt{3}$ (2) $4 + \sqrt{3}$ (3) $2 + \sqrt{3}$ (4) $1.5 + \sqrt{3}$

$$3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots}}}} \text{ का मान बराबर है :}$$

- (1) $3 + 2\sqrt{3}$ (2) $4 + \sqrt{3}$ (3) $2 + \sqrt{3}$ (4) $1.5 + \sqrt{3}$

Ans. (4)

Sol. Let $y = 3 + \frac{1}{4 + \frac{1}{y}}$

$$y = 3 + \frac{y}{4y+1}$$

$$\Rightarrow 4y^2 + y = 12y + 3 + y$$

$$\Rightarrow 4y^2 - 12y - 3 = 0$$

$$\Rightarrow y = \frac{12 \pm \sqrt{144 + 48}}{8}$$

$$\Rightarrow y = \frac{12 \pm 8\sqrt{3}}{8}$$

$$\Rightarrow y = \frac{3 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow y = 1.5 \pm \sqrt{3}$$

$$y = 1.5 + \sqrt{3}.$$

Indefinite Integration

13. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}} dx$ is equal to:

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5} + c$

(2) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5} + c$

(3) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5} + c$

(4) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5} + c$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

समाकलन $\int \frac{(2x-1) \cos \sqrt{(2x-1)^2 + 5}}{\sqrt{4x^2 - 4x + 6}} dx$ बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\frac{1}{2} \sin \sqrt{(2x+1)^2 + 5} + C$

(2) $\frac{1}{2} \sin \sqrt{(2x-1)^2 + 5} + C$

(3) $\frac{1}{2} \cos \sqrt{(2x+1)^2 + 5} + C$

(4) $\frac{1}{2} \cos \sqrt{(2x-1)^2 + 5} + C$

Ans. (2)

Sol. $\int \frac{(2x-1) \cos \sqrt{(2x-1)^2 + 5}}{\sqrt{(2x-1)^2 + 5}} dx$

Put $(2x-1)^2 + 5 = t^2$

$2(2x-1) dx = 2tdt$

$\Rightarrow \int \frac{\cos t}{t} \times \frac{t}{2} dx = \frac{1}{2} \sin t + C$

$= \frac{1}{2} \sin \sqrt{(2x-1)^2 + 5} + C$

Differential equation

14. The differential equations satisfied by the system of parabolas $y^2 = 4a(x+a)$ is:

(1) $y \left(\frac{dy}{dx} \right) + 2x \left(\frac{dy}{dx} \right) - y = 0$

(2) $y \left(\frac{dy}{dx} \right)^2 + 2x \left(\frac{dy}{dx} \right) - y = 0$

(3) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) - y = 0$

(4) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) + y = 0$

निम्न में से कौन सा अवकल समीकरण परवलयों के निकाय $y^2 = 4a(x+a)$ द्वारा संतुष्ट होता है ?

(1) $y \left(\frac{dy}{dx} \right) + 2x \left(\frac{dy}{dx} \right) - y = 0$

(2) $y \left(\frac{dy}{dx} \right)^2 + 2x \left(\frac{dy}{dx} \right) - y = 0$

(3) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) - y = 0$

(4) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) + y = 0$

Ans. (2)

Sol. $y^2 = 4a(x + a) \quad \dots\dots(1)$

$2yy' = 4a$

$\therefore yy' = 2a$

\therefore by(1) $y^2 = 2yy' \left(x + \frac{yy'}{2} \right)$

$y^2 = 2yy'x + (yy')^2$

$\Rightarrow y(y')^2 + 2xyy' - y = 0$

(as $y \neq 0$)

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Inverse Trigonometric Functions

- 15.** The real valued function $f(x) = \frac{\cosec^{-1}x}{\sqrt{x-[x]}}$, where $[x]$ denotes the greatest integer less than or equal to x , is defined for all x belonging to:
- all non-integers except the interval $[-1, 1]$
 - all integers except $0, -1, 1$
 - all reals except integers
 - all reals except the interval $[-1, 1]$

वास्तविक मान फलन $f(x) = \frac{\cosec^{-1}x}{\sqrt{x-[x]}}$, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है, का प्रांत है :

- (1) अंतराल $[-1, 1]$ के अतिरिक्त सभी अपूर्णांक
- (2) $0, -1, 1$ के अतिरिक्त सभी पूर्णांक
- (3) पूर्णांकों के अतिरिक्त सभी वास्तविक संख्याएँ
- (4) अंतराल $[-1, 1]$ के अतिरिक्त सभी वास्तविक संख्याएँ

Ans. (1)

Sol. $f(x) = \frac{\cosec^{-1}x}{\sqrt{x-[x]}}$

$$x \in (-\infty, -1] \cup [1, \infty)$$

$$\& \{x\} \neq 0$$

$$x \neq \text{Integer}$$

$$\Rightarrow x \in (-\infty, -1) \cup (1, \infty) - \text{all integers}$$

Limits

- 16.** If $\lim_{x \rightarrow 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3}$ is equal to L , then the value of $(6L + 1)$ is:

- (1) $\frac{1}{2}$
- (2) 2
- (3) $\frac{1}{6}$
- (4) 6

यदि $\lim_{x \rightarrow 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3} = L$ है, तो $(6L + 1)$ का मान है :

- (1) $\frac{1}{2}$
- (2) 2
- (3) $\frac{1}{6}$
- (4) 6

Ans. (2)

Sol. $L = \lim_{x \rightarrow 0} \frac{\left(x + \frac{x^3}{6} + \dots\right) - \left(x - \frac{x^3}{3} \dots\right)}{3x^3}$

$$L = \frac{1}{3} \left(\frac{1}{6} + \frac{1}{3} \right) = \frac{1}{6}$$

$$\Rightarrow 6L + 1 = 6 \cdot \frac{1}{6} + 1 = 2$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Circle

17. For all four circles M, N, O and P, following four equations are given:

$$\text{Circle M : } x^2 + y^2 = 1$$

$$\text{Circle N : } x^2 + y^2 - 2x = 0$$

$$\text{Circle O : } x^2 + y^2 - 2x - 2y + 1 = 0$$

$$\text{Circle P : } x^2 + y^2 - 2y = 0$$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a:

- (1) Rectangle (2) Square (3) Parallelogram (4) Rhombus

चार वृत्तों M, N, O तथा P के समीकरण हैं :

$$\text{वृत्त M : } x^2 + y^2 = 1$$

$$\text{वृत्त N : } x^2 + y^2 - 2x = 0$$

$$\text{वृत्त O : } x^2 + y^2 - 2x - 2y + 1 = 0$$

$$\text{वृत्त P : } x^2 + y^2 - 2y = 0$$

यदि वृत्त M का केन्द्र वृत्त N के केन्द्र से मिलाया जाता है, वृत्त N का केन्द्र वृत्त O के केन्द्र से मिलाया जाता है, वृत्त O का केन्द्र वृत्त P के केन्द्र से मिलाया जाता है तथा वृत्त P का केन्द्र वृत्त M के केन्द्र से मिलाया जाता है, तो ये रेखाएँ निम्न में से किस की भुजाएँ हैं ?

- (1) आयत (2) वर्ग (3) समांतर चतुर्भुज (4) समचतुर्भुज

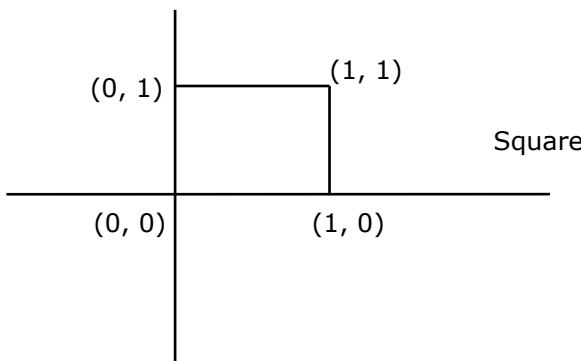
Ans. (2)

Sol. $C_M = (0, 0)$

$$C_N = (1, 0)$$

$$C_O = (1, 1)$$

$$C_P = (0, 1)$$



Binomial Theorem

18. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + \dots + a_{37}$ is equal to:

- (1) $2^{20}(2^{20} + 21)$ (2) $2^{19}(2^{20} + 21)$ (3) $2^{20}(2^{20} - 21)$ (4) $2^{19}(2^{20} - 21)$

माना $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$ है। तो $a_1 + a_3 + a_5 + \dots + a_{37}$ बराबर है :

- (1) $2^{20}(2^{20} + 21)$ (2) $2^{19}(2^{20} + 21)$ (3) $2^{20}(2^{20} - 21)$ (4) $2^{19}(2^{20} - 21)$

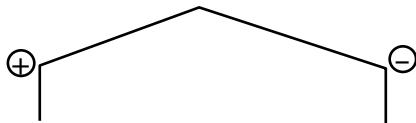
Ans. (4)

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\text{Sol. } \tan \tan^{-1} \sqrt{2} = \left| \frac{m - 3\sqrt{2}}{1 + 3m\sqrt{2}} \right|$$

$$\sqrt{2} = \left| \frac{m - 3\sqrt{2}}{1 + 3m\sqrt{2}} \right|$$



$$6m + \sqrt{2} = m - 3\sqrt{2}$$

$$5m = -4\sqrt{2}$$

$$m = -\frac{4\sqrt{2}}{5}$$

$$-6m - \sqrt{2} = m - 3\sqrt{2}$$

$$2\sqrt{2} = 7m$$

$$m = \frac{2\sqrt{2}}{7}$$

SECTION – B

Permutation Combination

1. The numbers of times all digit 3 will be written when listing the integers from 1 to 1000 is _____.

1 से 1000 तक के पूर्णांकों को क्रम से लिखने पर अंक 3, _____ बार लिखा जायेगा।

Ans. (300)

$$\text{Sol. } \begin{array}{ccccccccc} \boxed{3} & & \frac{10}{\uparrow} & & \frac{10}{\uparrow} & + & \frac{9}{\uparrow} & \boxed{\underline{3}} & \frac{10}{\uparrow} & + & \frac{9}{\uparrow} & \frac{10}{\uparrow} & \boxed{\underline{3}} \\ & & \uparrow & & \uparrow & & & \uparrow & & & & \uparrow & & \uparrow \end{array}$$

$$\Rightarrow 100 + 90 + 90$$

$$\Rightarrow 280$$

$$\left(\frac{10}{\uparrow} \right) + \left(\frac{9}{\uparrow} \right) \Rightarrow \boxed{19}$$

$$3 \rightarrow 1$$

$$280 + 19 + 1 = 300$$

3-D

2. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is _____.

समतल $x - 2y + 2z - 3 = 0$ के समातर तथा बिंदु $(1, 2, 3)$ से इकाई दूरी पर समतलों के समीकरण $ax + by + cz + d = 0$ हैं। यदि $(b - d) = K(c - a)$, तो k का धनात्मक मान है _____।

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Ans. (4)

Sol. $x - 2y + 2z + \lambda = 0$

Now given

$$d = \frac{|1 - 4 + 6 + \lambda|}{\sqrt{9}} = 1$$

$$|\lambda + 3| = 3$$

$$\lambda + 3 = \pm 3 \Rightarrow \lambda = 0, -6$$

So planes are: $x - 2y + 2z - 6 = 0$

$$x - 2y + 2z = 0$$

$$b - d = -2 + 6 = 4$$

$$c - a = 2 - 1 = 1$$

$$\Rightarrow \frac{b - d}{c - a} = k \Rightarrow k = 4$$

Definite Integration

3. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^4 f(x^2) dx$ is _____.

माना $f(x)$ तथा $g(x)$ दो फलन हैं, जो $f(x^2) + g(4 - x) = 4x^3$ तथा $g(4 - x) + g(x) = 0$ को संतुष्ट करते हैं। तो

$$\int_{-4}^4 f(x^2) dx \text{ का मान है } _____ |$$

Ans. (512)

Sol. $I = 2 \int_0^4 f(x^2) dx \quad \dots\dots\dots(1)$

$$\Rightarrow I = 2 \int_0^4 f((4 - x)^2) dx \quad \dots\dots\dots(2)$$

Adding equation (1) & (2)

$$2I = 2 \int_0^4 [f(x)^2 + f(4 - x)^2] dx \quad \dots\dots\dots(3)$$

$$\text{Now using } f(x^2) + g(4 - x) = 4x^3 \quad \dots\dots\dots(4)$$

$$x \rightarrow 4 - x$$

$$f((4 - x)^2) + g(x) = 4(4 - x)^3 \quad \dots\dots\dots(5)$$

Adding equation (4) & (5)

$$f(x^2) + f(4 - x^2) + g(x) + g(4 - x) = 4(x^3 + (4 - x)^3)$$

$$\Rightarrow f(x^2) + f(4 - x^2) = 4(x^3 + (4 - x)^3)$$

$$\text{Now, } I = 4 \int_0^4 x^3 + (4 - x)^3 dx = 512$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Central tendency & differentiation

4. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is _____.
 एक स्कूल में 25 अध्यापकों की औसत आयु 40 वर्ष है। एक अध्यापक 60 वर्ष की आयु में सेवानिवृत्त होता है तथा उसकी जगह एक नये अध्यापक की नियुक्ति होती है। यदि अब इस स्कूल में अध्यापकों की औसत आयु 39 वर्ष है, तो नये नियुक्त किए गए अध्यापक की आयु (वर्षों में) है _____।

Ans. (35)

Sol. $x_1 + x_2 + \dots + x_{25} = 25 \times 40 = 1000$

$$\frac{x_1 + x_2 + \dots + x_{25} - 60 + a}{25} = 39$$

$$100 - 60 + a = 25 \times 39$$

$$a = -940 + 975$$

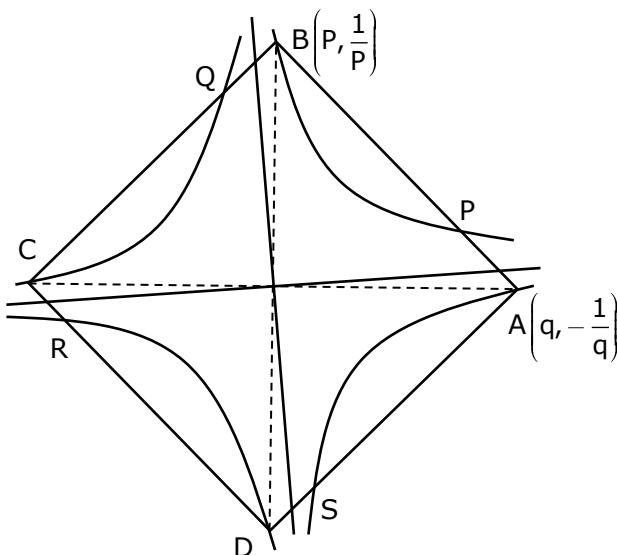
$$a = 35$$

Area Under the Curve

5. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is _____.
 एक वर्ग ABCD के सभी शीर्ष वक्र $x^2y^2 = 1$ पर है। इसकी भुजाओं के मध्यबिंदु भी इसी वक्र पर हैं तो ABCD के क्षेत्रफल का वर्ग है _____।

Ans. (80)

Sol.



Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$OA \perp OB$

$$\Rightarrow \left(\frac{1}{P^2} \right) \left(-\frac{1}{Q^2} \right) = -1$$

$$\Rightarrow P^2 Q^2 = 1$$

$$P \left(\frac{p+q}{2}, \frac{\frac{1}{p} - \frac{1}{q}}{2} \right) \text{ lies}$$

On $x^2 - y^2 = 1$

$$\Rightarrow (p+q)^2 \left(\frac{1}{p} - \frac{1}{q} \right)^2 = 16$$

$$\Rightarrow (p+q)^2 (p-q)^2 = 16$$

$$\Rightarrow (p^2 - q^2)^2 = 16$$

$$\Rightarrow P^2 - \frac{1}{P^2} = \pm 4$$

$$\Rightarrow p^4 \pm 4p^2 - 1 = 0$$

$$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$$

$$\Rightarrow p^2 = 2 + \sqrt{5} \text{ or } -2 + \sqrt{5}$$

$$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2 + \sqrt{5}} \text{ or } -2 + \sqrt{5} + \frac{1}{-2 + \sqrt{5}} = 2\sqrt{5}$$

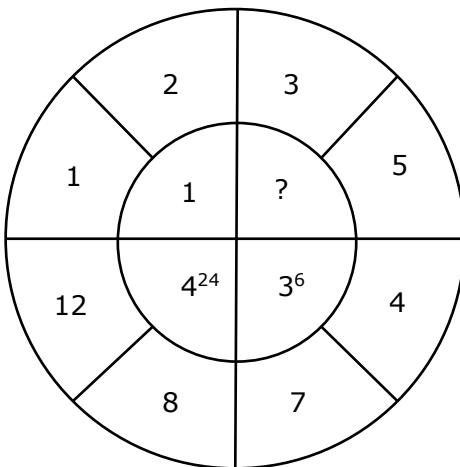
$$\text{Area} = 4 \left(\frac{1}{2} \right) (OA)(OB) = 2(OB)^2 = 4\sqrt{5}$$

Toll Free : 1800-212-1799

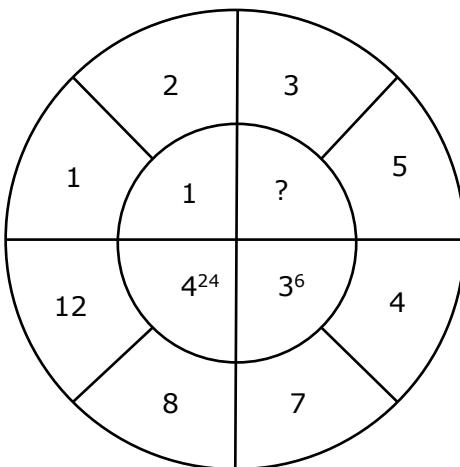
www.motion.ac.in | Email : info@motion.ac.in

Mathematics reasoning

6. The missing value in the following figure is _____.



निम्न चित्र में अनुपस्थित मान है _____।



Ans. (4)

Sol. 4^{24} has base 4 ($= 12 - 8$)

36 has base 3 ($= 7 - 4$)

(?) will have base 2 ($= 5 - 3$)

Power $24 = 6 \times 4 = (\text{no. of divisor of } 12) \times (\text{no. of divisor of } 8)$

Power $6 = 2 \times 3 = (\text{no. of divisor of } 7) \times (\text{no. of divisor of } 4)$

(?) will have power $= (\text{no. of divisor of } 3) \times (\text{no. of divisor of } 5) = 2 \times 2 = 4$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

Function

7. The numbers of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is _____.

अंतराल $[0, 2\pi]$ में समीकरण $|\cot x| = \cot x + \frac{1}{\sin x}$ के हलों की संख्या है _____।

Ans. (1)

Sol. Case I : $x \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

$$\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \text{not possible}$$

Case II : $x \in \left[\frac{\pi}{2}, \pi\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

$$-\cot x = \cot x + \frac{1}{\sin x}$$

$$\Rightarrow \frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\Rightarrow \cos x = -\frac{1}{2}$$

$$\Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3} = 1$$

Tangents Normals

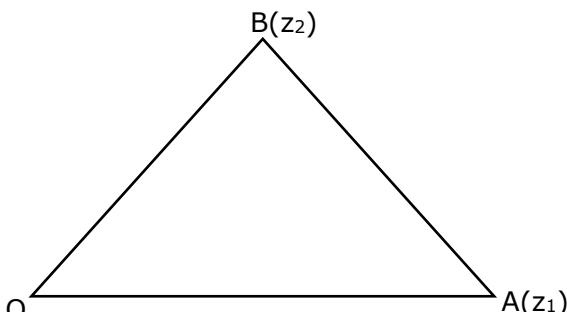
8. Let z_1, z_2 be the roots of the equations $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is _____.

माना समीकरण $z^2 + az + 12 = 0$ के मूल z_1 तथा z_2 हैं तथा मूलबिंदु के साथ z_1, z_2 एक समबाहु त्रिभुज बनाते हैं तो $|a|$ का मान है _____।

Ans. (6)

Sol. In equilateral Δ ,

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$



$$z_1^2 + z_2^2 = z_1 z_2$$

$$\therefore z_3 = 0$$

$$(z_1 + z_2)^2 = 3z_1 z_2$$

$$a^2 = 36$$

$$|a| = 6$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

3-D

9. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

माना बिंदुओं $(4, -3, 1)$ तथा $(2, 3, -5)$ को मिलाने वाली रेखा को समतल $ax + by + cz + d = 0$ समकोण पर समद्विभाजित करता है। यदि a, b, c, d पूर्णांक हैं तो $(a^2 + b^2 + c^2 + d^2)$ का न्यूनतम मान है _____।

Ans. (28)

Sol. normal of plane = \vec{PQ}

$$= -2\hat{i} + 6\hat{j} - 6\hat{k}$$

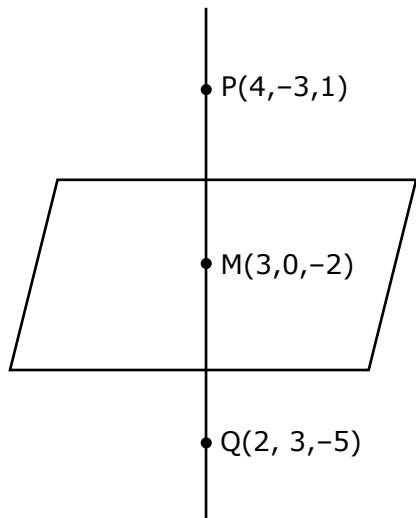
$$a = -2, b = 6, c = -6$$

& equation of plane is

$$-2x + 6y - 6z + d = 0$$

$$\downarrow M(3, 0, -2)$$

$$d = -6$$



Now equation of plane is

$$-2x + 6y - 6z - 6 = 0$$

$$x - 3y + 3z + 3 = 0$$

$$\Rightarrow (a^2 + b^2 + c^2 + d^2)_{\min} = 1^2 + 9 + 9 + 9 = 28$$

Indefinite Integration

10. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$), $f(0) = 0$ and $f(1) = \frac{1}{k}$, then the value of K is _____.

यदि $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$), $f(0) = 0$ तथा $f(1) = \frac{1}{k}$ है, तो K मान है _____।

Ans. (4)

Sol. $\int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = \int \frac{5x^8 + 7x^6}{x^{14} \left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

$$\int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$$

$$\text{put } 2 + \frac{1}{x^5} + \frac{1}{x^7} = t$$

$$\Rightarrow -\left(\frac{5}{x^6} + \frac{7}{x^8}\right) dx = dt$$

$$\int \frac{-dt}{t^2} = \frac{1}{t} + C$$

$$\Rightarrow f(x) = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C = \frac{x^7}{2x^7 + 1 + x^2} + C$$

$$f(0) = 0 \Rightarrow C = 0$$

$$f(x) = \frac{1}{4} = \frac{1}{k}$$

$$\Rightarrow k = 4$$

Toll Free : 1800-212-1799

www.motion.ac.in | Email : info@motion.ac.in

रिपिटर्स बैच का सर्वश्रेष्ठ परिणाम
सिर्फ मोशन के साथ

MOTION™

Another opportunity to
strengthen your preparation

UNNATI CRASH COURSE

JEE Main May 2021
at Kota Classroom

- ◆ **40 Classes** of each subjects
- ◆ **Doubt Clearing sessions by Expert faculties**
- ◆ **Full Syllabus Tests** to improve your question solving skills
- ◆ Thorough learning of concepts with regular classes
- ◆ **Get tips & trick** along with sample papers

Course Fee : ₹ 20,000



Start your **JEE Advanced 2021**
Preparation with

UTTHAN CRASH COURSE

at Kota Classroom

- ◆ Complete course coverage
- ◆ **55 Classes** of each subject
- ◆ **17 Full & 6 Part syllabus tests** will strengthen your exam endurance
- ◆ **Doubt clearing sessions** under the guidance of expert faculties
- ◆ **Get tips & trick** along with sample papers

Course Fee : ₹ 20,000

