



**JEE
MAIN
MARCH
2021**

**17th March 2021 | Shift - 1
MATHEMATICS**

JEE | NEET | Foundation

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SELECTIONS SINCE 2007

SECTION – A

Differential equation

1. Which of the following is true for $y(x)$ that satisfies the differential equation

$$\frac{dy}{dx} = xy - 1 + x - y; y(0) = 0 :$$

(1) $y(1) = 1$

(2) $y(1) = e^{\frac{1}{2}} - 1$

(3) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(4) $y(1) = e^{-\frac{1}{2}} - 1$

$y(x)$, जो अवकल समीकरण $\frac{dy}{dx} = xy - 1 + x - y; y(0) = 0$ को सन्तुष्ट करता है, के लिए निम्न में से कौन सा सत्य है ?

(1) $y(1) = 1$

(2) $y(1) = e^{\frac{1}{2}} - 1$

(3) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(4) $y(1) = e^{-\frac{1}{2}} - 1$

Ans. (4)

Sol. $\frac{dy}{dx} = (x - 1)y + (x - 1)$

$$\frac{dy}{dx} = (x - 1)(y + 1)$$

$$\frac{dy}{y+1} = (x-1)dx$$

$$\ln(y+1) = \frac{x^2}{2} - x + C$$

$$x = 0, y = 0 \Rightarrow C = 0 \quad \therefore \ln(y+1) = \frac{x^2}{2} - x$$

$$\text{putting } x = 1, \ln(y+1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

..

$$y = e^{-\frac{1}{2}} - 1$$

$$\therefore y(1) = e^{-\frac{1}{2}} - 1$$

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DETERMINANT

2. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution if k is equal to:

- (1) -2
- (2) -1
- (3) 1
- (4) 0

समीकरण निकाय $kx + y + z = 1$, $x + ky + z = k$ तथा $x + y + zk = k^2$ का कोई हल नहीं है यदि k बराबर है :

- (1) -2
- (2) -1
- (3) 1
- (4) 0

Ans. (1)

Sol. $D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$

$$\begin{aligned} \Rightarrow k(k^2 - 1) - (k - 1) + (1 - k) &= 0 \\ \Rightarrow (k - 1)(k^2 + k - 1 - 1) &= 0 \\ \Rightarrow (k - 1)(k^2 + k - 2) &= 0 \\ \Rightarrow (k - 1)(k - 1)(k + 2) &= 0 \\ \Rightarrow k = 1, k = 2 & \end{aligned}$$

for $k = 1$ equation identical so $k = -2$ for no solution.

Progressions

3. The value of $4 + \frac{1}{5 + \frac{1}{4 + \frac{1}{5 + \frac{1}{4 + \dots, \infty}}}}$ is:

- (1) $2 + \frac{4}{\sqrt{5}} \sqrt{30}$
- (2) $4 + \frac{4}{\sqrt{5}} \sqrt{30}$
- (3) $2 + \frac{2}{5} \sqrt{30}$
- (4) $5 + \frac{2}{5} \sqrt{30}$

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$$4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \dots \dots \infty}}}}$$

का मान है :

(1) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$

(2) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$

(3) $2 + \frac{2}{5}\sqrt{30}$

(4) $5 + \frac{2}{5}\sqrt{30}$

Ans. (3)

Sol. $y = 4 + \frac{1}{5 + \frac{1}{y}}$

$$\Rightarrow y = 4 + \frac{y}{5y + 1}$$

$$\Rightarrow 5y^2 - 20y - 4 = 0$$

$$\Rightarrow y = \frac{20 \pm \sqrt{400 + 80}}{10}$$

$$\Rightarrow y = \frac{20 \pm 4\sqrt{30}}{10}, y > 0$$

$$y = \frac{10 + 2\sqrt{30}}{5}$$

Mathematics reasoning

4. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to:
- (1) $p \Rightarrow \sim q$
 - (2) $p \Rightarrow q$
 - (3) $q \Rightarrow p$
 - (4) $\sim q \Rightarrow p$

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यदि बूलीय व्यंजक $(p \Rightarrow q) \Leftrightarrow (q * (\sim P))$ एक पुनरुक्ति है, तो बूलीय व्यंजक $p * (\sim q)$ किस के तुल्य है :

- (1) $p \Rightarrow \sim q$ (2) $p \Rightarrow q$ (3) $q \Rightarrow p$ (4) $\sim q \Rightarrow p$

Ans. (3)

Sol. $(p \rightarrow q) \Leftrightarrow (q * \sim P)$

P	q	$p \rightarrow q$	$q * \sim P$	$\sim q$	$\sim q \wedge p$	$\sim(\sim q \wedge p)$
T	T	T	T	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	F	T

$$\therefore \sim(\sim q \wedge p) = q \vee \sim p \\ = \sim p \vee q$$

$\therefore *$ is equivalent to \vee

$$\therefore p * \sim q = p \vee \sim q \\ = \sim q \vee p \\ = q \rightarrow p$$

Circle

5. Choose the incorrect statement about the two circles whose equations are given below:

$$x^2 + y^2 - 10x - 10y + 41 = 0 \text{ and}$$

$$x^2 + y^2 - 16x - 10y + 80 = 0$$

(1) Distance between two centres is the average of radii of both the circles.

(2) Circles have two intersection points.

(3) Both circles' centres lie inside region of one another.

(4) Both circles pass through the centre of each other.

दो वृत्तों :

$$x^2 + y^2 - 10x - 10y + 41 = 0 \text{ तथा}$$

$$x^2 + y^2 - 16x - 10y + 80 = 0$$

के लिए असत्य कथन चुनिए :

(1) दो केन्द्रों के बीच की दूरी दोनों वृत्तों की त्रिज्याओं का माध्य है

(2) वृत्तों के दो प्रतिच्छेदन बिंदु हैं

(3) दो वृत्तों के केन्द्र एक दूसरे के आंतरिक भाग में हैं

(4) दोनों वृत्त एक दूसरे के केन्द्र से होकर जाते हैं

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Ans. (3)

Sol. $C_1(5, 5), C_2(8, 5)$

position of $C_1(5, 5)$ in $S_2 = 0$

$$= 25 + 25 - 80 - 50 + 80$$

$$= 0$$

position of $C_2(8, 5)$ in $S_1 = 0$

$$\equiv 64 + 25 - 80 - 50 + 41$$

$$= 0$$

Inverse Trigonometric Functions

6. The sum of possible values of x for $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is:

(1) $-\frac{33}{4}$

(2) $-\frac{32}{4}$

(3) $-\frac{31}{4}$

(4) $-\frac{30}{4}$

$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ के लिए x के संभव मानों का योगफल है :

(1) $-\frac{33}{4}$

(2) $-\frac{32}{4}$

(3) $-\frac{31}{4}$

(4) $-\frac{30}{4}$

Ans. (2)

Sol. Taking tan both sides

$$\frac{(1+x)+(x-1)}{1-(1+x)(x-1)} = \frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 4x^2 + 31x - 8 = 0$$

$$\Rightarrow x = -8, \frac{1}{4}$$

$$\text{but at } x = \frac{1}{4}$$

$$\text{LHS} > \frac{\pi}{2} \text{ and RHS} < \frac{\pi}{2}$$

So, only solution is $x = -8$

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VECTOR

7. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :

- (1) 10 (2) 13 (3) 12 (4) 8

माना $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ तथा $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$

यदि $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$ है, तो $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ बराबर है :

- (1) 10 (2) 13 (3) 12 (4) 8

Ans. (3)

Sol. $\vec{a} = (2, -3, 4)$, $\vec{b} = (7, 1, -6)$

$$\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$$

$$\vec{r} \times (\vec{a} - \vec{b}) = 0$$

$$\vec{r} = \lambda(\vec{a} - \vec{b})$$

$$\vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$$

$$\vec{r} \cdot (2, -3, 1) = ?$$

$$\text{if } \vec{r} \cdot (1, 2, 1) = -3$$

$$\lambda(-5 - 8 + 10) = -3 \Rightarrow \lambda = 1$$

$$\therefore (-5, -4, 10) \cdot (2, -3, 1)$$

$$\Rightarrow -10 + 12 + 10 = 12$$

3-D

8. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is:

- (1) $3x + z = 6$
 (2) $3x - z = 0$
 (3) $x + 3z = 10$
 (4) $x + 3z = 0$

बिंदु (1, 2, 3) से होकर जाने वाले समतल, जिसमें y-अक्ष स्थित है, का समीकरण है :

- (1) $3x + z = 6$
 (2) $3x - z = 0$
 (3) $x + 3z = 10$
 (4) $x + 3z = 0$

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Ans. (2)

Sol. Let the equation of the plane is $a(x - 1) + b(y - 2) + c(z - 3) = 0$

Y-axis lies on it D.R.'s of y-axis are 0, 1, 0

$$\therefore 0.a + 1.b + 0.c = 0 \Rightarrow b = 0$$

$$\therefore \text{Equation of plane is } a(x - 1) + c(z - 3) = 0$$

$$x = 0, z = 0 \text{ also satisfy it } -a - 3c = 0 \Rightarrow a = -3c$$

$$-3c(x - 1) + c(z - 3) = 0$$

$$-3x + 3 + z - 3 = 0$$

$$3x - z = 0$$

MATRIX

9. If $A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is:

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

यदि $A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$ तथा $\det\left(A^2 - \frac{1}{2}I\right) = 0$ है, तो α का एक संभव मान है :

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (4)

$$\text{Sol. } A^2 = \begin{bmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{bmatrix} \begin{bmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{bmatrix} = \begin{bmatrix} \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha \end{bmatrix}$$

$$A^2 - \frac{1}{2}I = \begin{bmatrix} \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \sin^2 \alpha - \frac{1}{2} & 0 \\ 0 & \sin^2 \alpha - \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \left| A^2 - \frac{1}{2}I \right| = 0$$

$$\Rightarrow \begin{vmatrix} \sin^2 \alpha - \frac{1}{2} & 0 \\ 0 & \sin^2 \alpha - \frac{1}{2} \end{vmatrix} = 0$$

$$\Rightarrow \left(\sin^2 \alpha - \frac{1}{2} \right)^2 = 0 \Rightarrow \sin^2 \alpha = \frac{1}{2} \quad \Rightarrow \sin \alpha = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$$

..

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Circle

- 10** The line $2x-y+1=0$ is a tangent to the circle at the point (2,5) and the centre of the circle lies on $x-2y=4$. Then, the radius of the circle is:

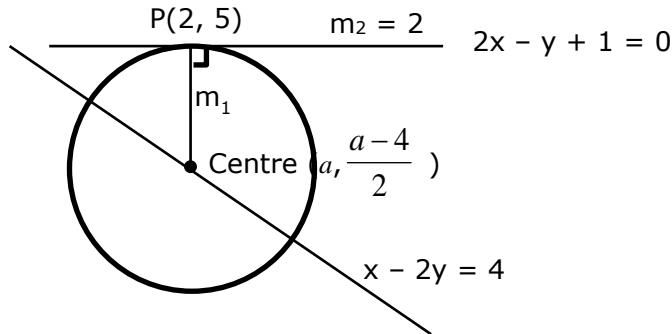
- (1) $4\sqrt{5}$
- (2) $5\sqrt{3}$
- (3) $3\sqrt{5}$
- (4) $5\sqrt{4}$

एक वृत्त के बिंदु (2, 5) पर स्पर्श रेखा का समीकरण $2x-y+1=0$ है तथा वृत्त का केन्द्र रेखा $x-2y=4$ पर है। तो वृत्त की त्रिज्या है :

- (1) $4\sqrt{5}$
- (2) $5\sqrt{3}$
- (3) ..
- (4) $5\sqrt{4}$

Ans. (3)

Sol.



$$m_1 \times m_2 = -1$$

$$\frac{\frac{a-4}{2}-5}{a-2} \times 2 = -1$$

$$\frac{a-14}{a-2} = -1$$

$$a-14 = 2-a$$

$$2a = 16$$

$$a = 8$$

$$\therefore \text{Centre } (8, 2)$$

$$\begin{aligned} \text{Radius} &= \sqrt{36+9} = \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

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Permutation Combination

11. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to:

- (1) 5
- (2) 6
- (3) 2
- (4) 4

टीम 'A' में 7 लड़के तथा n लड़कियाँ हैं तथा टीम 'B' में 4 लड़के तथा 6 लड़कियाँ हैं। यदि इन दो टीम के बीच कुल 52 एकल मैच आयोजित किए जा सकते हैं, जब एक लड़का, एक लड़के के विरुद्ध खेलता है तथा एक लड़की, एक लड़की के विरुद्ध खेलती है, तो n बराबर है :

- (1) 5
- (2) 6
- (3) 2
- (4) 4

Ans. (4)

Sol.

$$7 \times 4 + 6 \times n = 52$$

$$6n = 24$$

$$\Rightarrow n = 4$$

Solution of Triangle

12. In a triangle PQR, the co-ordinates of the points P and Q are (-2, 4) and (4, -2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the $\triangle PQR$ is:

- (1) (-2, -2)
- (2) (0, 2)
- (3) (-1, 0)
- (4) (1, 4)

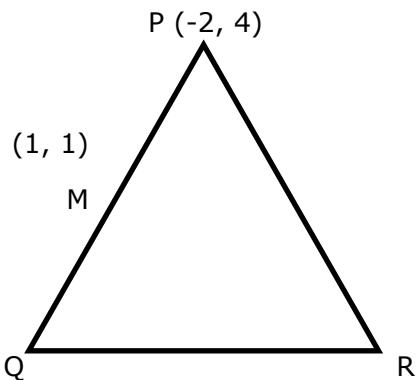
एक त्रिभुज PQR में, बिंदुओं P तथा Q के निर्देशांक क्रमशः (-2, 4) तथा (4, -2) हैं। यदि PR के लम्ब समद्विभाजक का समीकरण $2x - y + 2 = 0$ है, तो $\triangle PQR$ के परिवृत का केन्द्र है :

- (1) (-2, -2)
- (2) (0, 2)
- (3) (-1, 0)
- (4) (1, 4)

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Ans. (1)



Sol. (4, -2)

Perpendicular bisector of PR : $2x - y + 2 = 0 \dots\dots(1)$

Mid-point of PQ \rightarrow M(1, 1)

Equation of perpendicular bisector of PQ : $x - y = 0 \dots\dots(2)$

\therefore POI of equation (1) & (2) is circumcentre

So, circumcentre (-2, -2)

Inverse Trigonometric Functions

13. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots\dots$ upto 100 terms, then α is:

- (1) 1.03 (2) 1.00 (3) 1.01 (4) 1.02

यदि $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots\dots$ 100 पदों तक, तो α बराबर है :

- (1) 1.03 (2) 1.00 (3) 1.01 (4) 1.02

Ans. (3)

$$\text{Sol. } \text{RHS} = \sum_{n=1}^{100} \cot^{-1} 2n^2 = \sum_{n=1}^{100} \tan^{-1} \left(\frac{2}{4n^2} \right)$$

$$= \sum_{n=1}^{100} \tan^{-1} \left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)} \right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$= \tan^{-1} 201 - \tan^{-1} 1$$

$$= \tan^{-1} \left(\frac{200}{202} \right)$$

$$\Rightarrow \cot^{-1} \alpha = \cot^{-1} \left(\frac{101}{100} \right)$$

$$\Rightarrow \alpha = 1.01$$

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Monotonicity

14. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in \mathbb{R}$ such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$$

- (1) $g(\alpha)$ is a strictly decreasing function
- (2) $g(\alpha)$ has an inflection point at $\alpha = -\frac{1}{2}$
- (3) $g(\alpha)$ is an even function
- (4) $g(\alpha)$ is a strictly increasing function.

फलन $g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$, के लिए $\alpha \in \mathbb{R}$ के लिए निम्न में से कौन सा कथन असत्य है ?

- (1) $g(\alpha)$ एक निरंतर व्यापक फलन है
- (2) $g(\alpha)$ का $\alpha = -\frac{1}{2}$ पर एक नति परिवर्तन बिंदु है
- (3) $g(\alpha)$ एक सम फलन है
- (4) $g(\alpha)$ एक निरंतर वर्धमान फलन है

Ans. (1 OR 2 OR 3/Bonus)

Sol. $g(x) = \frac{\pi}{12}$ by applying prop. $x \rightarrow a + b - x$.

Binomial Theorem

15. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in \mathbb{N}$ is equal to:

- (1) 4
- (2) 3
- (3) 2
- (4) 1

यदि $(x + x^{\log_2 x})^7$ के प्रसार में चौथा पद 4480 है, तो x ($x \in \mathbb{N}$) का मान है :

- (1) 4
- (2) 3
- (3) 2
- (4) 1

Ans. (3)

Sol. ${}^7 C_3 x^4 (x^{\log_2 x})^3 = 4480 \Rightarrow 35x^4 (x^{\log_2 x})^3 = 4480 \Rightarrow x^4 (x^{\log_2 x})^3 = 128$

take log w.r.t base 2 we get $4\log_2 x + 3\log_2 (x^{\log_2 x}) = \log_2 128$

Let $\log_2 x = y \Rightarrow 4y + 3y^2 = 7$

$$\Rightarrow y = 1, \frac{-7}{3} \Rightarrow \log_2 x = 1, \frac{-7}{3}$$

$$x = 2, x = 2^{-7/3}$$

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Probability

16. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is:

(1) $\frac{17}{36}$

(2) $\frac{4}{9}$

(3) $\frac{5}{12}$

(4) $\frac{1}{2}$

दो पासें फेंके जाते हैं। यदि दोनों पासों के छः तलों (faces) पर अंकित संख्याएँ 1, 2, 3, 5, 7 तथा 11 हैं तो ऊपर के तलों पर प्रकट होने वाली संख्याओं का योगफल ≤ 8 होने की प्रायिकता है :

(1) $\frac{17}{36}$

(2) $\frac{4}{9}$

(3) $\frac{5}{12}$

(4) $\frac{1}{2}$

Ans. (1)

Sol. $n(S) = 36$

possible ordered pair ; (1, 1), (1, 2), (1, 3), (1, 5), (1, 7), (2, 1), (2, 2), (2, 3), (2, 5), (3, 1), (3, 2), (3, 3), (3, 5), (5, 1), (5, 2), (5, 3), (7, 1)

Number of ordered pair = 17

$$\text{Probability} = \frac{17}{36}$$

Function

17. The inverse of $y = 5^{\log x}$ is:

(1) $x = 5^{\log y}$

(2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$

(4) $x = 5^{\frac{1}{\log y}}$

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$y = 5^{\log x}$ का प्रतिलोम है :

(1) $x = 5^{\log y}$

(2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$

(4) $x = 5^{\frac{1}{\log y}}$

Ans. (3)

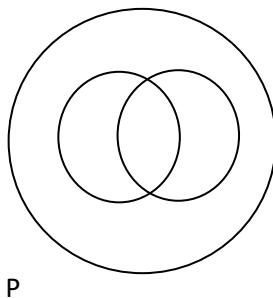
Sol. $y = 5^{\log_e x}$

$$\log_5 y = \log_e x$$

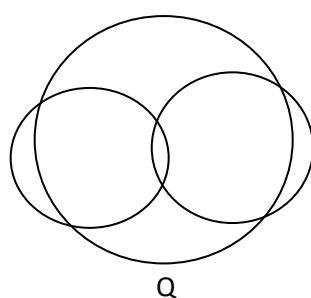
$$x = e^{\log_5 y}$$

Probability

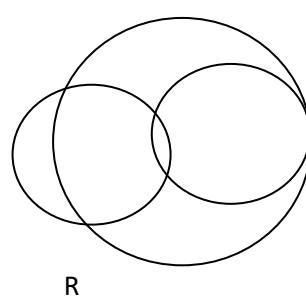
18. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statements.



P



Q



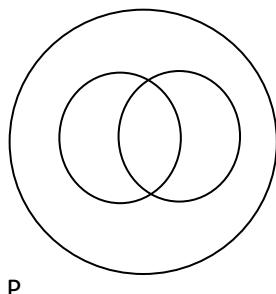
R

- (1) P and R
- (2) P and Q
- (3) None of these
- (4) Q and R

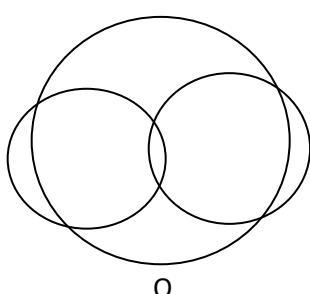
एक स्कूल में तीन तरह के खेल खेले जाते हैं। कुछ छात्र दो तरह के खेल खेलते हैं, परन्तु कोई भी सभी तीन खेल नहीं खेलता है। उपर्युक्त कथन को कौन से वेन आरेख दर्शाती है ?

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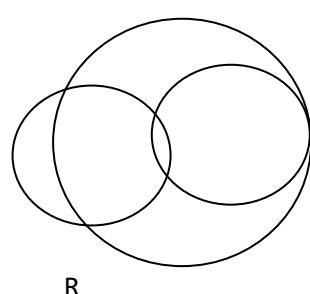
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P



Q



R

- (1) P तथा R
- (2) P तथा Q
- (3) इनमें से कोई नहीं
- (4) Q तथा R

Ans. (3)

Sol. In all the (A), (B), (C) there are some students which play all the three games hence no venn diagram is correct.

Complex Number

19. The area of the triangle with vertices A(z), B(iz) and C(z+iz) is:

- (1) $\frac{1}{2}|z + iz|^2$
- (2) 1
- (3) $\frac{1}{2}$
- (4) $\frac{1}{2}|z|^2$

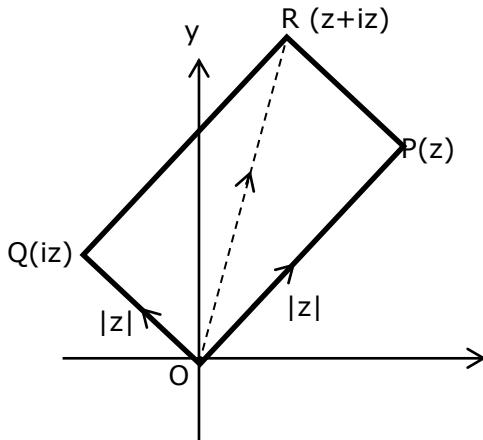
शीर्ष A(z), B(iz) तथा C(z+iz) वाले त्रिभुज का क्षेत्रफल है :

- (1) $\frac{1}{2}|z + iz|^2$
- (2) 1
- (3) $\frac{1}{2}$
- (4) $\frac{1}{2}|z|^2$

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Ans. (4)



Sol.

$$\text{Area of } \Delta = \frac{1}{2}(\text{area of square}) = \frac{1}{2}|z|^2$$

Limits

20. The value of $\lim_{x \rightarrow 0^+} \frac{\cos^{-1}(x - [x]^2) \cdot \sin^{-1}(x - [x]^2)}{x - x^3}$, where $[x]$ denotes the greatest integer $\leq x$ is :

(1) 0

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{2}$

(4) π

$$\lim_{x \rightarrow 0^+} \frac{\cos^{-1}(x - [x]^2) \cdot \sin^{-1}(x - [x]^2)}{x - x^3}, \text{ जहाँ } [x] \text{ महत्तम पूर्णांक } \leq x \text{ है, का मान है :}$$

(1) 0

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{2}$

(4) π

Ans. (3)

$$\lim_{x \rightarrow 0^+} \frac{\cos^{-1}(x - [x]^2) \sin^{-1}(x - [x]^2)}{x(1 - x^2)}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{\cos^{-1} x \sin^{-1} x}{x} = \frac{\pi}{2}$$

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SECTION - B

Probability

1. Let there by three independent events E_1 , E_2 and E_3 . The probability that only E_1 occurs is α , only E_2 occurs is β and only E_3 occurs is γ . Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0, 1)$

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to _____.

माना E_1 , E_2 तथा E_3 स्वतंत्र घटनायें हैं। केवल E_1 के घटित होने की प्रायिकता α है, केवल E_2 के घटित होने की प्रायिकता β है तथा केवल E_3 के घटित होने की प्रायिकता γ है। माना किसी भी घटना के न घटने की प्रायिकता 'p' है, जो समीकरणों $(\alpha - 2\beta)p = \alpha\beta$ तथा $(\beta - 3\gamma)p = 2\beta\gamma$ को सतुर्ण करता है। सभी दी गई प्रायिकताएँ अंतराल $(0, 1)$ में हैं। तो $\frac{E_1 \text{ के घटित होने की प्रायिकता}}{E_3 \text{ के घटित होने की प्रायिकता}} \text{ बराबर है } _____$ ।

Ans. (6)

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$$\Rightarrow x(1-y)(1-z) = \alpha \Rightarrow y(1-x)(1-z) = \beta \Rightarrow z(1-x)(1-y) = \gamma \Rightarrow (1-x)(1-y)(1-z) = P$$

Putting in the given relation we get $x = 2y$ and $y = 3z \Rightarrow x = 6z \Rightarrow \frac{x}{z} = 6$

3-D

2. If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0$, $3x-5y+4z+11=0$ and the point $(-2, 1, 3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is _____.

यदि समतलों $2x-7y+4z-3=0$, $3x-5y+4z+11=0$ की प्रतिच्छेदन रेखा तथा बिंदु $(-2, 1, 3)$ से होकर जाने वाले समतल का समीकरण $ax+by+cz-7=0$ है, तो $2a+b+c-7$ का मान है _____।

Ans. (4)

Sol.

Equation of plane can be written using family of planes: $P_1 + \lambda P_2 = 0$

$$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$$

It passes through $(-2, 1, 3)$

$$\therefore (-4 + 7 + 12 - 3) + \lambda(-6 - 5 + 12 + 11) = 0$$

$$-2 + \lambda(12) = 0$$

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$$\lambda = \frac{1}{6}.$$

$$\therefore 12x - 42y + 24z - 18 + 3x - 5y + 4z + 11 = 0$$

$$15x - 47y + 28z - 7 = 0$$

$$a = 15, b = -47, c = 28$$

$$\therefore 2a + b + c - 7 = 30 - 47 + 28 - 7$$

$$= 4$$

[Ans : 4]

MATRIX

3. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - \text{Adj}(2A))^{10}$ is equal to _____.

यदि $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ है, तो $\det(A^4) + \det(A^{10} - \text{Adj}(2A))^{10}$ का मान बराबर है _____।

Ans. (16)

Sol. $|A| = -2 \Rightarrow |A|^4 = 16$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10}-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2 \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$(\text{adj}(2A))^{10} = 2^{10} \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$$

$$= 2^{10} \begin{bmatrix} 1 & -(2^{10}-1) \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - (\text{adj}(2A))^{10} = \begin{bmatrix} 0 & 2^{11} \times 1023 \\ 0 & 1 - (1024)^2 \end{bmatrix}$$

$$|A^{10} - \text{adj}(2A)^{10}| = 0$$

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Circle

4. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 one the other circle for the given circles' equations

$$x^2 + y^2 - 10x - 10y + 41 = 0$$

$$x^2 + y^2 - 24x - 10y + 160 = 0 \quad \text{_____}.$$

वृत्तों

$$x^2 + y^2 - 10x - 10y + 41 = 0 \text{ तथा}$$

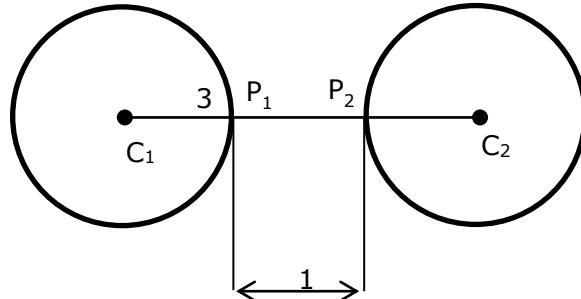
$$x^2 + y^2 - 24x - 10y + 160 = 0$$

के लिए यदि बिंदु P_1 एक वृत्त पर है तथा बिंदु P_2 दूसरे वृत्त पर है, तो बिंदुओं P_1 तथा P_2 के बीच की न्यूनतम दूरी है _____ |

Ans. (1)

Sol. $S_1 : (x - 5)^2 + (y - 5)^2 = 9$ centre $(5, 5)$, $r_1 = 3$

$S_2 : (x - 12)^2 + (y - 5)^2 = 9$ centre $(12, 5)$, $r_2 = 3$



$$\text{So } (P_1P_2)_{\min} = 1$$

Binomial Theorem

5. If $(2021)^{3762}$ is divided by 17, then the remainder is _____.
 $(2021)^{3762}$ को 17 से विभाजित करने पर शेषफल है _____ |

Ans. (4)

Sol. $(2021)^{3762} = (2023 - 2)^{3762} = \text{multiple of } 17 + 2^{3762}$
 $= 17\lambda + 2^2 (2^4)^{940}$
 $= 17\lambda + 4 (17 - 1)^{940}$
 $= 17\lambda + 4 (17\mu + 1)$
 $17k + 4; (k \in I)$
Remainder = 4

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Definite Integration

6. If $[\cdot]$ represents the greatest integer function, then the value of $\left| \int_0^{\frac{\pi}{2}} [x^2] - \cos x \, dx \right|$ is _____.

यदि $[\cdot]$ महत्तम पूर्णांक फलन को दर्शाता है, तो $\left| \int_0^{\frac{\pi}{2}} [x^2] - \cos x \, dx \right|$ का मान है _____।

Ans. (1)

Sol.

$$\begin{aligned}
 & \int_0^{\frac{\pi}{2}} [x^2] - \cos x \, dx \\
 &= \int_0^1 [-\cos x] \, dx + \int_1^{\frac{\pi}{2}} [1 - \cos x] \, dx \\
 &= \int_0^1 -1 - [\cos x] \, dx + \int_1^{\frac{\pi}{2}} dx + \int_1^{\frac{\pi}{2}} -1 - [\cos x] \, dx \\
 &= -\int_0^1 dx + \int_1^{\frac{\pi}{2}} dx - \int_1^{\frac{\pi}{2}} dx \\
 &= -(x)_0^1 = -1
 \end{aligned}$$

$\therefore \boxed{\text{Ans : 1}}$

Maxima & Minima

7. यदि $f(x) = \sin \left(\cos^{-1} \left(\frac{1-2^{2x}}{1+2^{2x}} \right) \right)$ है तथा इसका x के सापेक्ष प्रथम अवकलज $-\frac{b}{a} \log_e 2$ है जब $x = 1$, जहाँ a तथा b पूर्णांक हैं, तो $|a^2 - b^2|$ का न्यूनतम मान है _____।

If $f(x) = \sin \left(\cos^{-1} \left(\frac{1-2^{2x}}{1+2^{2x}} \right) \right)$ and its first derivative with respect to x is $-\frac{b}{a} \log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _____.

Ans. (481)

Sol.

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$$\cos^{-1} \left(\frac{1 - 4^x}{1 + 4^x} \right) \quad \text{Let } 2^x = t > 0$$

$$\cos^{-1} \left(\frac{1 - t^2}{1 + t^2} \right) \quad t > 0$$

$$\cos^{-1} (\cos 2\theta) \quad t = \tan \theta$$

$$\theta \in \left(0, \frac{\pi}{2} \right)$$

$$2\theta \in (0, \pi)$$

$$\Rightarrow 2\theta$$

$$\therefore \sin \left\{ \cos^{-1} \left(\frac{1 - 4^x}{1 + 4^x} \right) \right\}$$

$$= \sin 2\theta$$

$$y = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2t}{1 + t^2} = \frac{2 \cdot 2^x}{1 + 4^x}$$

$$\therefore \frac{dy}{dx} = \frac{(1 + 4^x) \cdot 2 \cdot 2^x \ln 2 - 2 \cdot 2^x (4^x) \cdot \ln 4}{(1 + 4^x)^2}$$

$$\frac{dy}{dx} = \frac{5 \cdot 2 \cdot 2 \cdot \ln 2 - 2 \cdot 2 \cdot 4 \cdot 2 \ln 2}{25}$$

$$= \frac{20 \ln 2 - 32 \ln 2}{25} = \frac{-12 \ln 2}{25}$$

$$a = 25, b = 12$$

$$\therefore |a^2 - b^2|_{\min} = |25^2 - 12^2|$$

$$= (481)$$

Continuity

8. If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$,

then k is _____.

यदि फलन $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$, अपने प्रांत के प्रत्येक बिंदु पर संतत है तथा $f(0) = \frac{1}{k}$ है, तो k बराबर है _____।

Ans. (6)

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Sol. $\frac{1}{k} = \lim_{x \rightarrow 0} \frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right)}{x^4}$

$$= \lim_{x \rightarrow 0} \frac{(\sin x + x)(x - \sin x)}{2x^4} = \left(\lim_{x \rightarrow 0} \frac{\sin x + x}{2x} \right) \left(\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \right)$$

$$= 1 \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} = \frac{1}{6}$$

Area Under the Curve

9. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \leq 100$ and $4x + 3y \leq 75$ for $x \geq 0$ and $y \geq 0$ is _____.

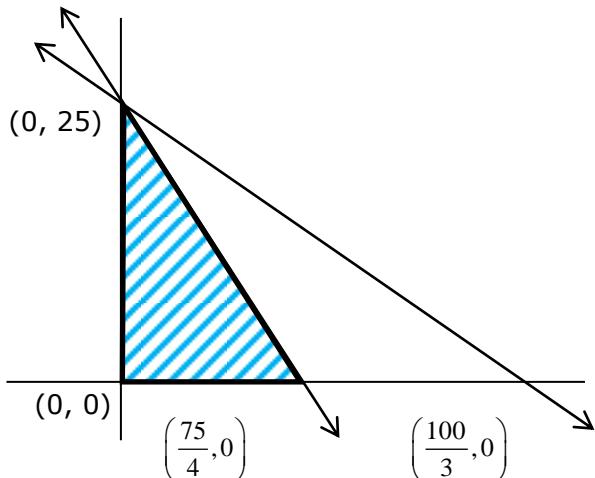
$z = 6xy + y^2$ अधिकतम मान, जबकि $3x + 4y \leq 100$ तथा $4x + 3y \leq 75$, $x \geq 0$ and $y \geq 0$ है, है _____।

Ans. (625)

Sol. $x \geq 0, y \geq 0$

$$4x + 3y \leq 75$$

$$3x + 4y \leq 100$$



(x, y)	$z = 6xy + y^2$
$(0, 25)$	$z = 0 + 625 = 625$
$\left(\frac{75}{4}, 0\right)$	$z = 0 + 0 = 0$
$(0, 0)$	$z = 0 + 0 = 0$
$z_{\max} = 625$ at $x = 0, y = 25$	

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VECTOR

10. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$,

$$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k} \text{ and}$$

$$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$$

such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ is equal to _____.

$$\text{यदि } \vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k},$$

$$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k} \text{ तथा}$$

$$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$$

है, जिनके लिए $\vec{a} \cdot \vec{b} = 1$ तथा $\vec{b} \cdot \vec{c} = -3$ है, तो $\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ बराबर है _____।

Ans. (2)

Sol.

$$\vec{a} = (\alpha, \beta, 3) \quad \vec{b} = (-\beta, -\alpha, -1)$$

$$\vec{c} = (1, -2, -1)$$

$$\bullet \vec{a} \cdot \vec{b} = 1 \Rightarrow (-\alpha\beta - \alpha\beta - 3) = 1$$

$$-2\alpha\beta = 4$$

$$\boxed{\alpha\beta = -2}$$

$$\bullet \vec{b} \cdot \vec{c} = -3$$

$$-\beta + 2\alpha + 1 = -3$$

$$\boxed{2\alpha - \beta = -4} \quad \therefore \begin{cases} \alpha = -1 \\ \beta = 2 \end{cases}$$

$$\vec{a} = (-1, 2, 3) \quad \vec{b} = (-2, 1, -1)$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & 1 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(7) + \hat{k}(3) = (-5, -7, 3)$$

$$\therefore (\vec{a} \times \vec{b}) \cdot \vec{c} = (-5, -7, 3) \cdot (1, -2, -1)$$

$$= (-5 + 14 - 3) = 6$$

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