



**JEE
MAIN
MARCH
2021**

**16thMarch 2021 | Shift - 1
Mathematics**

JEE | NEET | Foundation

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SELECTIONS SINCE 2007

SECTION -A

Topics:- Mathematics reasoning

1. Consider three observations a , b and c such that $b = a+c$. If the standard deviation of $a+2$, $b+2$, $c+2$ is d , then which of the following is true?
- (1) $b^2 = a^2 + c^2 + 3d^2$ (2) $b^2 = 3(a^2 + c^2) - 9d^2$
 (3) $b^2 = 3(a^2 + c^2) + 9d^2$ (4) $b^2 = 3(a^2 + c^2 + d^2)$
1. तीन प्रेक्षणों a , b तथा c का विचार किजिए, जिनके लिए $b = a+c$ है। यदि $a+2$, $b+2$, $c+2$ का मानक विचलन d है, तो निम्न में से कौन सा सत्य है ?
- (1) $b^2 = a^2 + c^2 + 3d^2$ (2) $b^2 = 3(a^2 + c^2) - 9d^2$
 (3) $b^2 = 3(a^2 + c^2) + 9d^2$ (4) $b^2 = 3(a^2 + c^2 + d^2)$

Ans. (2)

Sol. for a , b , c

$$\text{mean} = \bar{x} = \frac{a+b+c}{3}$$

$$\bar{x} = \frac{2b}{3}$$

S.D. of a , b , c = d

$$d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$$

$$b^2 = 3a^2 + 3c^2 - 9d^2$$

VECTOR

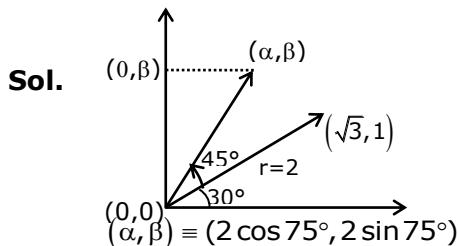
2. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counter clockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to :

- (1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

2. माना मूल बिन्दु के सापेक्ष सदिश $\sqrt{3}\hat{i} + \hat{j}$ को प्रथम चतुर्थांश दिशा में वामावर्त दिशा में 45° के कोण तक घुमाने पर सदिश $\alpha\hat{i} + \beta\hat{j}$ प्राप्त होता है। तो शीर्ष (α, β) , $(0, \beta)$ तथा $(0, 0)$ के त्रिभुज का क्षेत्रफल बराबर है:

- (1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

Ans. (2)



$$\text{Area} = \frac{1}{2} (2 \cos 75^\circ)(2 \sin 75^\circ)$$

$$= \sin(150^\circ) = \frac{1}{2} \text{ square unit}$$

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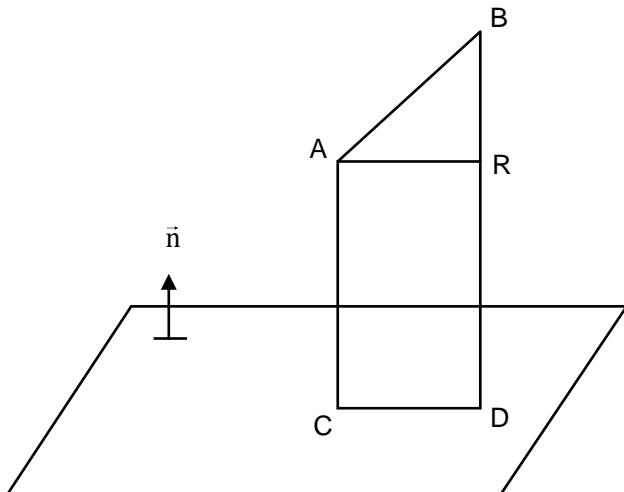
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3-D

3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :
 (1) $\sqrt{41}$ (2) $\sqrt{55}$ (3) $\sqrt{31}$ (4) $\sqrt{66}$
3. यदि $a > 0$ के लिए, बिन्दुओं $A(a, -2a, 3)$ तथा $B(0, 4, 5)$ से समतल $lx + my + nz = 0$ पर लम्बों के पाद क्रमशः बिन्दु $C(0, -a, -1)$ तथा D हैं, तो रेखा खंड CD की लम्बाई है :
 (1) $\sqrt{41}$ (2) $\sqrt{55}$ (3) $\sqrt{31}$ (4) $\sqrt{66}$

Ans. (4)

Sol.



$$CD = AR = |AB| \sin\phi$$

$$CD = |AB| \sqrt{1 - \cos^2 \phi}$$

$$|AB| \sqrt{1 - \left(\frac{\overrightarrow{AB} \cdot \vec{n}}{|\overrightarrow{AB}|} \right)^2}$$

$$= \sqrt{(AB)^2 - (\overrightarrow{AB} \cdot \vec{n})^2}$$

$$\cos \phi = \frac{\overrightarrow{AB} \cdot \vec{n}}{|\vec{n}| |\overrightarrow{AB}|}$$

$$|\overrightarrow{AB}| = a\hat{i} - (2a+4)\hat{j} - 2\hat{k}$$

$$\overrightarrow{AB} \cdot \vec{n} = \ell a - (2a+4) - 2n$$

C on plane

$$0\ell - am - n = 0 \quad \dots (1)$$

$$\overrightarrow{AC} \parallel \vec{n}$$

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$$\frac{a}{\ell} = \frac{-a}{m} = \frac{4}{n}$$

$$m = -\ell \text{ & } an + 4m = 0 \quad \dots (2)$$

From (1) and (2)

$$a^2m + an = 0$$

$$\underline{4m + an = 0}$$

$$(a^2 - 4)m = 0 \Rightarrow \boxed{a = 2} .$$

$$2m + n = 0 \quad \dots (1)$$

$$m + \ell = 0$$

$$\ell^2 + m^2 + n^2 = 1$$

$$m^2 + m^2 + 4m^2 = 1$$

$$m^2 = \frac{1}{6}$$

$$m = \frac{1}{\sqrt{6}}$$

$$n = \frac{-2}{\sqrt{6}}$$

$$\ell = \frac{-1}{\sqrt{6}}$$

$$\begin{aligned} \text{Now } \vec{AB} \cdot \vec{n} &= 2\left(\frac{-1}{\sqrt{6}}\right) - 8\left(\frac{1}{\sqrt{6}}\right) - 2\left(\frac{-2}{\sqrt{6}}\right) \\ &= \frac{-2-8+4}{\sqrt{6}} = -\sqrt{6} \end{aligned}$$

$$|\vec{AB}| = \sqrt{4+64+4} = \sqrt{72}$$

$$CD = \sqrt{72-6}$$

$$CD = \sqrt{66}$$

Maxima & Minima

4. The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a-3)(x + \log_e 5) + 2(a-7) \cot\left(\frac{x}{2}\right) \sin^2\left(\frac{x}{2}\right), \quad x \neq 2n\pi, n \in \mathbb{N} \text{ has critical points, is :}$$

$$(1) \left[-\frac{4}{3}, 2 \right]$$

$$(2) [1, \infty)$$

$$(3) (-\infty, -1]$$

$$(4) (-3, 1)$$

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- 4.** $a \in R$ का परिसर, जिसके लिए फलन

$$f(x) = (4a-3)(x + \log_e 5) + 2(a-7) \cot\left(\frac{x}{2}\right) \sin^2\left(\frac{x}{2}\right), \quad x \neq 2n\pi, n \in \mathbb{N} \text{ के क्रांतिक बिन्दु हैं, हैं :}$$

- (1) $\left[-\frac{4}{3}, 2\right]$ (2) $[1, \infty)$ (3) $(-\infty, -1]$ (4) $(-3, 1)$

Ans. (1)

Sol. $f(x) = (4a - 3)(x + \ln 5) + 2(a - 7) \left(\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \cdot \sin^2 \frac{x}{2} \right)$

$$f(x) = (4a - 3)(x + \ln 5) + (a - 7) \sin x$$

$$f'(x) = (4a - 3) + (a - 7) \cos x = 0$$

$$\cos x = \frac{-(4a-3)}{a-7}$$

$$-1 \leq -\frac{4a-3}{a-7} < 1$$

$$-1 < \frac{4a - 3}{a - 7} \leq 1$$

$$\frac{4a-3}{a-7} - 1 \leq 0 \text{ and } \frac{4a-3}{a-7} + 1 > 0 \Rightarrow \frac{-4}{3} \leq a < 2$$

Differentiability

- 5.** Let the functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in R where $(f \circ g)(x)$ is NOT differentiable is equal to :

- 5.** माना फलन $f: R \rightarrow R$ तथा $g: R \rightarrow R$

$$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ तथा } g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

द्वारा परिभाषित हैं। तो R में उन बिन्दुओं की संख्या, जहाँ $(fog)(x)$ अवकलनीय नहीं है, है :

Ans. (1)

Sol. $fog(x) = \begin{cases} x^3 + 2, & x \leq 0 \\ x^6, & 0 \leq x \leq 1 \\ (3x - 2)^2, & x \geq 1 \end{cases}$

\therefore $f \circ g(x)$ is discontinuous at $x = 0$ then non-differentiable at $x = 0$

Now,

at $x = 1$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{(3(1+h) - 2)^2 - 1}{h} = 6$$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{(1-h)^6 - 1}{-h} = 6$$

Number of points of non-differentiability = 1

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Complex Number

Ans. (4)

$$\text{Sol. } \frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$$

$$2|z| + 22 \geq (|z| - 1)^2$$

$$2|z| + 22 \geq |z|^2 - 2|z| + 1$$

$$|z|^2 - 4|z| - 21 \leq 0$$

$$(|z| - 7)(|z| + 3) \leq 0$$

$$\Rightarrow |z| \leq 7$$

$$\therefore |z|_{\max} = 7$$

Probability

Ans. (3)

Sol. $P(\overline{S}_{\text{missing}} \text{ / both found spade}) = \frac{P(\overline{S}_m \cap \text{BFS})}{P(\text{BFS})}$

$$= \frac{\left(1 - \frac{13}{52}\right) \times \frac{13}{51} \times \frac{12}{50}}{}$$

$$= \left(1 - \frac{13}{52} \right) \times \frac{13}{51} \times \frac{12}{50} + \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}$$

$$= \frac{39}{50}$$

Binomial Theorem

- 8.** If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right)^{60}$, then $(n-1)$ is divisible by :

(1) 8 (2) 26 (3) 7 (4) 30

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Ans. (2)

Sol. $T_{r+1} = {}^{60} C_r \left(3^{1/4}\right)^{60-r} \left(5^{1/8}\right)^r$

rational if $\frac{60-r}{4}$, $\frac{r}{8}$, both are whole numbers, $r \in \{0,1,2,\dots,60\}$

$$\frac{60-r}{4} \in W \Rightarrow r \in \{0, 4, 8, \dots, 60\}$$

and $\frac{r}{8} \in W \Rightarrow r \in \{0, 8, 16, \dots, 56\}$

∴ Common terms $r \in \{0, 8, 16, \dots, 56\}$

So 8 terms are rational

Then Irrational terms = $61 - 8 = 53 = m$

$$\therefore n - 1 = 52 = 13 \times 2^2$$

factors 1,2,4,13,26,52

3-D

9. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$ respectively. Let lines PR and QS intersect at T. If the vector \overrightarrow{TA} is perpendicular to both \overrightarrow{PR} and \overrightarrow{QS} and the length of vector \overrightarrow{TA} is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{5}$ (2) $\sqrt{171}$ (3) $\sqrt{227}$ (4) $\sqrt{482}$

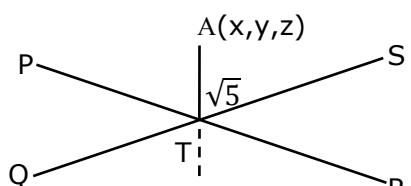
9. माना दो बिन्दुओं P तथा Q के स्थिति सदिश क्रमशः $3\hat{i} - \hat{j} + 2\hat{k}$ तथा $\hat{i} + 2\hat{j} - 4\hat{k}$ हैं। माना दो बिन्दु R तथा S इस प्रकार है कि रेखाओं PR तथा QS के दिक् अनुपात क्रमशः $(4, -1, 2)$ तथा $(-2, 1, -2)$ है। माना रेखाओं PR तथा QS का प्रतिच्छेदन बिन्दु T है। यदि सदिश \overrightarrow{TA} सदिशों \overrightarrow{PR} तथा \overrightarrow{QS} के लम्बवत् है तथा सदिश \overrightarrow{TA} की लम्बाई $\sqrt{5}$ इकाई है, तो A के एक स्थिति सदिश का मापांक है:

(1) $\sqrt{5}$ (2) $\sqrt{171}$ (3) $\sqrt{227}$ (4) $\sqrt{482}$

Ans. (2)

$$\text{Sol} \quad \vec{p} = 3\hat{i} - \hat{j} + 2\hat{k} \quad \& \quad \vec{\theta} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{v}_{PR} = \langle 4, -1, 2 \rangle \text{ & } \vec{v}_{OS} = \langle -2, 1, -2 \rangle$$



$$L_{PR} : \vec{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda \langle 4, -1, 2 \rangle$$

$$L_{QS} : \vec{r} = \langle \hat{i} + 2\hat{j} - 4\hat{k} \rangle + \mu \langle -2, 1, -2 \rangle$$

$$\text{Now } T \text{ on } PR = \langle 3 + 4\lambda, -1 - \lambda, 2 + 2\lambda \rangle$$

$$\text{Similarly } T \text{ on } QS = \langle 1 - 2\mu, 2 + \mu, -4 - 2\mu \rangle$$

$$\text{For } \lambda \text{ & } \mu: \left. \begin{array}{l} 3 + 4\lambda = 1 - 2\mu \Rightarrow \mu + 2\lambda = -1 \\ -1 - \lambda = 2 + \mu \Rightarrow \mu + \lambda = -3 \end{array} \right\} \begin{array}{l} \lambda = 2 \\ \mu = -5 \end{array}$$

$$2 + 2\lambda = -4 - 2\mu$$

$$\Rightarrow T : \langle 11, -3, 6 \rangle$$

$$\text{D.R. of TA} = \vec{v}_{QS} \times \vec{v}_{PR}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -2 \\ 4 & -1 & 2 \end{vmatrix} = 0\hat{i} - 4\hat{j} - 2\hat{k}$$

$$L_{TA} : \vec{r} = (11\hat{i} - 3\hat{j} + 6\hat{k}) + \lambda(-4\hat{j} - 2\hat{k})$$

$$\text{Now } A = \langle 11, -3 - 4\lambda, 6 - 2\lambda \rangle$$

$$TA = \sqrt{5}$$

$$(-3 + 4\lambda + 3)^2 + (6 + 2\lambda - 6)^2 = 5$$

$$16\lambda^2 + 4\lambda^2 = 5$$

$$20\lambda^2 = 5$$

$$\lambda = \pm \frac{1}{2}$$

$$A: (11, -3 - 2, 6 - 1) \quad ; \quad A: (11, -3 + 2, 6 + 1)$$

$$|A| = \sqrt{121 + 25 + 25} ; \quad |A| = \sqrt{121 + 1 + 49}$$

$$= \sqrt{171} \quad ; \quad \sqrt{171}$$

Parabola, Ellipse & Hyperbola

- 10.** If the three normals drawn to the parabola, $y^2=2x$ pass through the point $(a, 0)$ $a \neq 0$, then 'a' must be greater than:

- 10.** यदि परवलय $y^2 = 2x$ पर डाले गये तीन अभिलम्ब, बिन्दु $(a, 0)$ $a \neq 0$ से होकर जाते हैं, तो 'a' निम्न में से किस से अधिक होना चाहिए ?

Ans. (1)

Sol. Let the equation of the normal is

$$y = mx - 2am - am^3$$

$$\text{here } 4a = 2 \Rightarrow a = \frac{1}{2}$$

$$y = mx - m - \frac{1}{2}m^3$$

It passing through A(a, 0) then

$$0 = am - m - \frac{1}{2}m^3$$

$$m = 0, a - 1 - \frac{1}{2}m^2 = 0$$

$$m^2 = 2(a - 1) > 0$$

$$\therefore a > 1$$

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Trigonometry Phase-II

11. Let $S_k = \sum_{r=1}^k \tan^{-1} \left(\frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$. Then $\lim_{k \rightarrow \infty} S_k$ is equal to :

- (1) $\tan^{-1} \left(\frac{3}{2} \right)$ (2) $\cot^{-1} \left(\frac{3}{2} \right)$ (3) $\frac{\pi}{2}$ (4) $\tan^{-1}(3)$

11. माना $S_k = \sum_{r=1}^k \tan^{-1} \left(\frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$ है। तो $\lim_{k \rightarrow \infty} S_k$ बराबर है :

- (1) $\tan^{-1} \left(\frac{3}{2} \right)$ (2) $\cot^{-1} \left(\frac{3}{2} \right)$ (3) $\frac{\pi}{2}$ (4) $\tan^{-1}(3)$

Ans. (2)

$$\begin{aligned} \text{Sol. } & \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{6^r(3-2)}{\left(1 + \left(\frac{3}{2} \right)^{2r+1} \right) 2^{2r+1}} \right) \\ & \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{2^r \cdot 3^{r+1} - 3^r 2^{r+1}}{\left(1 + \left(\frac{3}{2} \right)^{2r+1} \right) 2^{2r+1}} \right) \\ & \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{\left(\frac{3}{2} \right)^{r+1} - \left(\frac{3}{2} \right)^r}{1 + \left(\frac{3}{2} \right)^{r+1} \left(\frac{3}{2} \right)^r} \right) = \sum_{r=1}^{\infty} \left[\tan^{-1} \left(\frac{3}{2} \right)^{r+1} - \tan^{-1} \left(\frac{3}{2} \right)^r \right] = \frac{\pi}{2} - \tan^{-1} \frac{3}{2} = \cot^{-1} \frac{3}{2} \end{aligned}$$

Trigonometry Phase-II

12. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

- (1) 3 (2) 2 (3) 4 (4) 8

12. अंतराल $[0, \pi]$ में समीकरण $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ के मूलों की संख्या है :

- (1) 3 (2) 2 (3) 4 (4) 8

Ans. (3)

$$(81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$$

$$(81)^{\sin^2 x} + \frac{81}{(81)^{\sin^2 x}} = 30$$

$$\text{Let } (81)^{\sin^2 x} = t$$

$$t + \frac{81}{t} = 30 \Rightarrow t^2 + 81 = 30t$$

$$t^2 - 30t + 81 = 0$$

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Mathematics reasoning

- 14.** Which of the following Boolean expression is a tautology?
 (1) $(p \wedge q) \wedge (p \rightarrow q)$ (2) $(p \wedge q) \vee (p \vee q)$ (3) $(p \wedge q) \vee (p \rightarrow q)$ (4) $(p \wedge q) \rightarrow (p \rightarrow q)$

14. निम्न में से कौन-सा बूलीय व्यंजक पुनरुक्ति है ?
 (1) $(p \wedge q) \wedge (p \rightarrow q)$ (2) $(p \wedge q) \vee (p \vee q)$ (3) $(p \wedge q) \vee (p \rightarrow q)$ (4) $(p \wedge q) \rightarrow (p \rightarrow q)$

Ans. (4)

Sol.	p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
	T	T	T	T	T	T
	F	T	F	T	T	T
	T	F	F	T	F	T
	F	F	F	F	T	T

MATRIX

Ans. (1)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 4 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$128 \begin{bmatrix} x - y \\ -x + y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \Rightarrow 128(x - y) = 8$$

$$\Rightarrow x - y = \frac{1}{16} \dots (1)$$

and $128(-x + y) = 64 \Rightarrow x - y = \frac{-1}{2} \dots (2)$

\Rightarrow no solution (from eq. (1) & (2))

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Log

Ans. (3)

Sol. $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$

$$\sin x \cos x = \frac{1}{10} \quad \dots(1)$$

$$\text{and } \log_{10} (\sin x + \cos x) = \frac{1}{2} (\log_{10} n - 1)$$

$$\Rightarrow \sin x + \cos x = \left(\frac{n}{10} \right)^{\frac{1}{2}}$$

$$\Rightarrow \sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{n}{10} \text{ (squaring)}$$

$$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10} \text{ (using equation(1))}$$

$$\Rightarrow \frac{n}{10} = \frac{12}{10} \Rightarrow n = 12$$

Parabola, Ellipse & Hyperbola

- 17.** The locus of the midpoints of the chord of the circle, $x^2+y^2=25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$ (2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$
 (3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$ (4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

17. वृत्त $x^2+y^2=25$ की उस जीवा, जो अति परवलय $\frac{x^2}{9} - \frac{y^2}{16} = 1$ की स्पर्श रेखा है, के मध्य बिन्दु का बिन्दुपथ है :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$ (2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$
 (3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$ (4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Ans. (4)

Sol. tangent of hyperbola

$$y = mx \pm \sqrt{9m^2 - 16} \quad \dots(i)$$

which is a chord of circle with mid-point (h, k)

so equation of chord T = S₁

$$hx + ky = h^2 + k^2$$

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$$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k} \quad \dots(ii)$$

by (i) and (ii)

$$m = -\frac{h}{k} \text{ and } \sqrt{9m^2 - 16} = \frac{h^2 + k^2}{k}$$

$$9 \frac{h^2}{k^2} - 16 = \frac{(h^2 + k^2)^2}{k^2}$$

$$\text{locus } 9x^2 - 16y^2 = (x^2 + y^2)^2$$

Binomial Theorem

- 18.** Let $[x]$ denote greatest integer less than or equal to x . If for $n \in N$, $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} \text{ is equal to :}$$

(1) 1

(2) n

(3) 2^{n-1}

(4) 2

- 18.** माना $[x]$ महत्तम पूर्णांक $\leq x$ है। यदि $n \in N$ के लिए $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$ हो, तो $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ बराबर है :

(1) 1

(2) n

(3) 2^{n-1}

(4) 2

Ans. (1)

Sol. $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$$(1-x+x^3)^n = a_0 + a_1x + a_2x^2 + \dots + a_{3n}x^{3n}$$

Put $x = 1$

$$1 = a_0 + a_1 + a_2 + a_3 + a_4 + \dots + a_{3n} \quad \dots(1)$$

Put $x = -1$

$$1 = a_0 - a_1 + a_2 - a_3 + a_4 \dots (-1)^{3n} a_{3n} \quad \dots(2)$$

Add (1) + (2)

$$\Rightarrow a_0 + a_2 + a_4 + a_6 + \dots = 1$$

Sub (1) - (2)

$$\Rightarrow a_1 + a_3 + a_5 + a_7 + \dots = 0$$

$$\text{Now } \sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$$

$$= (a_0 + a_2 + a_4 + \dots) + 4(a_1 + a_3 + \dots)$$

$$= 1 + 4 \times 0$$

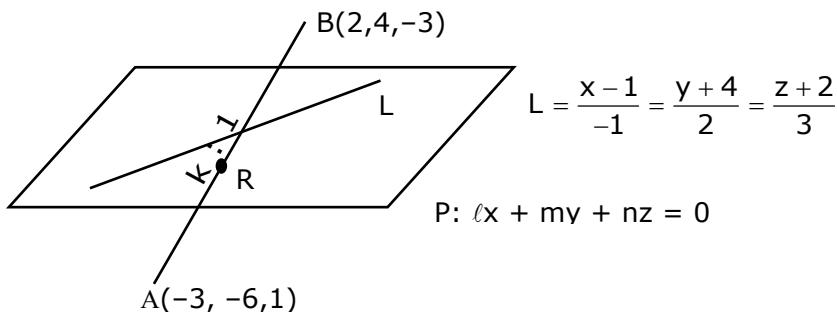
$$= 1$$

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3-D

19. Let P be a plane $\ell x + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(-3, -6, 1) and B(2, 4, -3) in ratio k : 1 then the value of k is equal to :
 (1) 1.5 (2) 2 (3) 4 (4) 3
19. माना P एक समतल $\ell x + my + nz = 0$ है, जिसमें रेखा $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$ स्थित है। यदि समतल P, बिन्दुओं A(-3, -6, 1) तथा B(2, 4, -3) को मिलाने वाले रेखाखंड AB को k : 1 के अनुपात बाँटता है, तो k का मान बराबर है :
 (1) 1.5 (2) 2 (3) 4 (4) 3

Ans. (2)

Sol.

Line lies on plane

$$-\ell + 2m + 3n = 0 \quad \dots(1)$$

Point on line (1, -4, -2) lies on plane

$$\ell - 4m - 2n = 0 \quad \dots(2)$$

from (1) & (2)

$$-2m + n = 0 \Rightarrow 2m = n$$

$$\ell = 3n + 2m \Rightarrow \ell = 4n$$

$$\ell : m : n :: 4n : \frac{n}{2} : n$$

$$\ell : m : n :: 8n : n : 2n$$

$$\ell : m : n :: 8 : 1 : 2$$

 Now equation of plane is $8x + y + 2z = 0$

R divide AB is ratio k : 1

$$R : \left(\frac{-3+2k}{k+1}, \frac{-6+4k}{k+1}, \frac{1-3k}{k+1} \right) \text{ lies on plane}$$

$$8\left(\frac{-3+2k}{k+1}\right) + \left(\frac{-6+4k}{k+1}\right) + 2\left(\frac{1-3k}{k+1}\right) = 0$$

$$-24 + 16k - 6 + 4k + 2 - 6k = 0$$

$$-28 + 14k = 0$$

$$k = 2$$

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Function

Ans. (1)

Sol. Case-1 $x \leq -4$

$$(-x - 3)(-x - 4) = 6$$

$$\Rightarrow (x + 3)(x + 4) = 6$$

$$\Rightarrow x^2 + 7x + 6 = 0$$

$$\Rightarrow x = -1 \text{ or } -6$$

but $x \leq -4$

$$x = -6$$

Case-2 $x \in (-4, 0)$

$$(-x - 3)(x + 4) = 6$$

$$\Rightarrow -x^2 - 7x - 12 - 6 = 0$$

$$\rightarrow x^2 + 7x + 18 = 0$$

$D < 0$ No solution

Case-3 $x \geq 0$

$$(x - 3)(x + 4) = 6$$

$$(x - 3)(x + 4) = 0$$

$$\Rightarrow x^2 + x - 18 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 72}}{2}$$

$$\therefore x = \frac{\sqrt{73} - 1}{2} \text{ only}$$

SECTION -B

Definite Integration

- 1.** Let $f: (0, 2) \rightarrow \mathbb{R}$ be defined as $f(x) = \log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right)$. Then, $\lim_{n \rightarrow \infty} \frac{2}{n} \left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f(1)\right)$ is equal to _____

1. माना $f: (0, 2) \rightarrow \mathbb{R}$ $f(x) = \log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right)$ द्वारा परिभाषित है। तो $\lim_{n \rightarrow \infty} \frac{2}{n} \left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f(1)\right)$ बराबर है

Ans. (1)

$$\text{Sol. } E = 2 \lim_{x \rightarrow \infty} \sum_{n=1}^{\infty} \frac{1}{n} f\left(\frac{r}{n}\right)$$

$$E = \frac{2}{\pi^2} \int_0^1 \ln \left(1 + \tan \frac{\pi x}{4} \right) dx \quad \dots(i)$$

replacing $x \rightarrow 1 - x$

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$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \tan \frac{\pi}{4} (1-x) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \tan \left(\frac{\pi}{4} - \frac{\pi}{4}x \right) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \frac{1 - \tan \frac{\pi}{4}x}{1 + \tan \frac{\pi}{4}x} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(\frac{2}{1 + \tan \frac{\pi x}{4}} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \left(\ln 2 - \ln \left(1 + \tan \frac{\pi x}{4} \right) \right) dx \quad \dots \text{(ii)}$$

equation (i) + (ii)

$$E = 1$$

MATRIX

2. The total number of 3×3 matrices A having entries from the set $\{0, 1, 2, 3\}$ such that the sum of all the diagonal entries of AA^T is 9, is equal to _____
 2. 3×3 के आव्यूहों A, जिनके अवयव समुच्चय $\{0, 1, 2, 3\}$ में से हैं तथा AA^T के विकर्ण के सभी अवयवों का योगफल 9 है, की कुल संख्या है _____।

Ans. (766)

$$\begin{aligned} \text{Sol. } AA^T &= \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x & a & d \\ y & b & e \\ z & c & f \end{bmatrix} \\ &= \begin{bmatrix} x^2 + y^2 + z^2 & ax + by + cz & dx + ey + fz \\ ax + by + cz & a^2 + b^2 + c^2 & ad + be + cf \\ dx + ey + fz & ad + be + cf & d^2 + e^2 + f^2 \end{bmatrix} \end{aligned}$$

$$\text{Tr}(AA^T) = x^2 + y^2 + z^2 + a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 9$$

all $\rightarrow 1$

$$\text{one 3, rest } = 0 \qquad \qquad \frac{9!}{8!} = 9$$

$$\text{two 2, one 1 & rest } 0 \qquad \qquad \frac{9!}{2!6!} = 63 \times 4 = 252$$

$$\text{one 2, five 1, rest } 0 \qquad \qquad \frac{9!}{5!3!} = 63 \times 8 = 504$$

= 766

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Definite Integration

3. Let $f: R \rightarrow R$ be a continuous function such that $f(x) + f(x+1) = 2$, for all $x \in R$. If $I_1 = \int_0^8 f(x) dx$ and $I_2 = \int_{-1}^3 f(x) dx$, then the value of $I_1 + 2I_2$ is equal to _____
3. माना $f: R \rightarrow R$ एक संतत फलन है जिसके लिए $f(x) + f(x+1) = 2$, $x \in R$ है। यदि $I_1 = \int_0^8 f(x) dx$ तथा $I_2 = \int_{-1}^3 f(x) dx$, हैं, तो $I_1 + 2I_2$ का मान बराबर है _____।
- Ans. (16)**
- Sol.** $f(x) + f(x+1) = 2 \dots \text{(i)}$
 $x \rightarrow (x+1)$
 $f(x+1) + f(x+2) = 2 \dots \text{(ii)}$
by (i) & (ii)
 $f(x) - f(x+2) = 0$
 $f(x+2) = f(x)$
 $f(x)$ is periodic with $T = 2$
 $I_1 = \int_0^{2 \times 4} f(x) dx = 4 \int_0^2 f(x) dx$
 $I_2 = \int_{-1}^3 f(x) dx = \int_0^4 f(x+1) dx = \int_0^4 (2 - f(x)) dx$
 $I_2 = 8 - 2 \int_0^2 f(x) dx$
 $I_1 + 2I_2 = 16$

Progressions

4. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____
4. एक समान्तर श्रेढ़ी तथा एक गुणोत्तर श्रेढ़ी के पहले चार पद समुच्चय $\{11, 8, 21, 16, 26, 32, 4\}$ में से हैं। यदि इन श्रेड़ियों अंतिम पद चार अंकों की अधिकतम सम्भव संख्यायें हैं, तो इन दोनों श्रेड़ियों में होने वाले पदों की संख्या है _____।

Ans. (3)

- Sol.** AP - 11, 16, 21, 26
GP - 4, 8, 16, 32
So common terms are 16, 256, 4096

Tangents Normals

5. If the normal to the curve $y(x) = \int_0^x (2t^2 - 15t + 10) dt$ at a point (a, b) is parallel to the line $x + 3y = -5$, $a > 1$, then the value of $|a + 6b|$ is equal to _____
5. यदि वक्र $y(x) = \int_0^x (2t^2 - 15t + 10) dt$ के बिन्दु (a, b) , $a > 1$, पर अभिलम्ब, रेखा $x + 3y = -5$ के समान्तर है, तो $|a + 6b|$ का मान बराबर है _____।

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Ans. (406)

Sol. $y'(x) = (2x^2 - 15x + 10)$

at point P

$$3 = (2a^2 - 15a + 10)$$

$$\Rightarrow 2a^2 - 15a + 7 = 0$$

$$\Rightarrow 2a^2 - 14a - a + 7 = 0$$

$$\Rightarrow 2a(a - 7) - 1(a - 7) = 0$$

$$a = \frac{1}{2} \text{ or } 7,$$

given $a > 1 \therefore a = 7$

also P lies on curve

$$\therefore b = \int_0^a (2t^2 - 15t + 10) dt$$

$$b = \int_0^7 (2t^2 - 15t + 10) dt$$

$$6b = -413$$

$$\therefore |a + 6b| = 406$$

Limits

6. If $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$, then $a+b+c$ is equal to _____

6. यदि $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$ है, तो $a+b+c$ बराबर है _____ ।

Ans. (4)

Sol.
$$\lim_{x \rightarrow 0} \frac{\left\{ a \left(1 + x + \frac{x^2}{2!} + \dots \right) - b \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \right) + c \left(1 - x + \frac{x^2}{2!} \dots \right) \right\}}{x \left(x - \frac{x^3}{3!} + \dots \right)} = 2$$

$$\therefore \lim_{x \rightarrow 0} \frac{(a - b + c) + x(a - c) + x^2 \left(\frac{a}{2} + \frac{b}{2} + \frac{c}{2} \right) + \dots}{x^2 \left(1 - \frac{x^2}{6} \dots \right)} = 2$$

$$\therefore a - b + c = 0$$

$$\& a - c = 0$$

$$\& \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = 2$$

$$\Rightarrow a + b + c = 4$$

Circle

7. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to _____

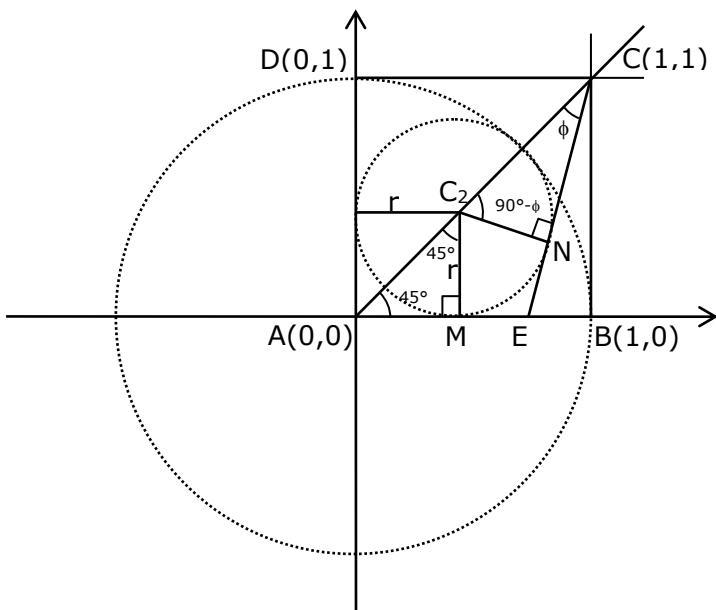
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7. माना भुजा की इकाई लम्बाई का एक वर्ग ABCD है। माना इकाई त्रिज्या तथा केन्द्र A का एक वृत्त C_1 खींचा जाता है। वृत्त C_1 तथा रेखाओं AD और AB को स्पर्श करता हुआ एक और वृत्त C_2 भी खींचा जाता है। माना बिन्दु C से वृत्त C_2 की एक स्पर्श रेखा भुजा AB को E पर मिलती है। यदि EB की लम्बाई $\alpha + \sqrt{3}\beta$ है, जहाँ α, β पूर्णांक हैं, तो $\alpha + \beta$ बराबर है _____।

Ans. (1)

Sol.



$$(i) \sqrt{2}r + r = 1$$

$$r = \frac{1}{\sqrt{2} + 1}$$

$$r = \sqrt{2} - 1$$

$$(ii) CC_2 = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

$$\text{From } \Delta CC_2N = \sin \phi = \frac{\sqrt{2} - 1}{2(\sqrt{2} - 1)}$$

$$\phi = 30^\circ$$

(iii) In ΔACE are sine law

$$\frac{AE}{\sin \phi} = \frac{AC}{\sin 105^\circ}$$

$$AE = \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{3} + 1} \cdot 2\sqrt{2}$$

$$AE = \frac{2}{\sqrt{3} + 1} = \sqrt{3} - 1$$

$$\therefore EB = 1 - (\sqrt{3} - 1)$$

$$2 - \sqrt{3}$$

$$\alpha = 2, \beta = -1 \Rightarrow \alpha + \beta = 1$$

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Complex Number

8. Let z and ω be two complex numbers such that $\omega = z\bar{z} - 2z + 2$, $\left| \frac{z+i}{z-3i} \right| = 1$ and $\operatorname{Re}(\omega)$ has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which ω^n is real, is equal to _____
8. माना दो सम्मिश्र संख्याओं z तथा ω के लिए $\omega = z\bar{z} - 2z + 2$, $\left| \frac{z+i}{z-3i} \right| = 1$ हैं तथा $\operatorname{Re}(\omega)$ का मान निम्नतम है। तो $n \in \mathbb{N}$ का निम्नतम मान, जिसके लिए ω^n वास्तविक है, बराबर है _____।

Ans. (4)

Sol. Let $z = x + iy$

$$|z + i| = |z - 3i|$$

$$\Rightarrow y = 1$$

$$\text{Now } \omega = x^2 + y^2 - 2x - 2iy + 2$$

$$\omega = x^2 + 1 - 2x - 2i + 2$$

$$\operatorname{Re}(\omega) = x^2 - 2x + 3$$

$$\operatorname{Re}(\omega) = (x-1)^2 + 2$$

$$\operatorname{Re}(\omega)_{\min} \text{ at } x = 1 \Rightarrow z = 1 + i$$

$$\text{Now } \omega = 1 + 1 - 2 - 2i + 2$$

$$\omega = 2(1-i) = 2\sqrt{2}e^{i\left(\frac{-\pi}{4}\right)}$$

$$\omega^n = 2\sqrt{2}e^{i\left(\frac{-n\pi}{4}\right)}$$

$$\text{If } \omega^n \text{ is real } \Rightarrow n = 4$$

MATRIX

9. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$ where $\omega = \frac{-1 + i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____

9. माना $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ तथा $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$ है, जहाँ $\omega = \frac{-1 + i\sqrt{3}}{2}$ है तथा 3 कोटि का तत्समक आव्यूह I_3 है। यदि आव्यूह $(P^{-1}AP - I_3)^2$ का सारणिक $\alpha\omega^2$ है, तो α का मान बराबर है _____।

Ans. (36)

Sol. $|P^{-1}AP - I|^2$

$$= |(P^{-1}AP - I)(P^{-1}AP - I)|^2$$

$$= |P^{-1}APP^{-1}AP - 2P^{-1}AP + I|^2$$

$$= |P^{-1}A^2P - 2P^{-1}AP + P^{-1}IP|^2$$

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$$= \left| P^{-1} (A^2 - 2A + I) P \right| = \left| P^{-1} (A - I)^2 P \right|$$

$$= \left| P^{-1} |A - I|^2 P \right| = |A - I|^2$$

$$= \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$$

$$= (1(\omega(\omega+1)+\omega) - 7\omega + \omega^2 \cdot \omega)^2$$

$$= (\omega^2 + 2\omega - 7\omega + 1)^2 = (\omega^2 - 5\omega + 1)^2$$

$$= (-6\omega)^2 = 36\omega^2 \Rightarrow \alpha = 36$$

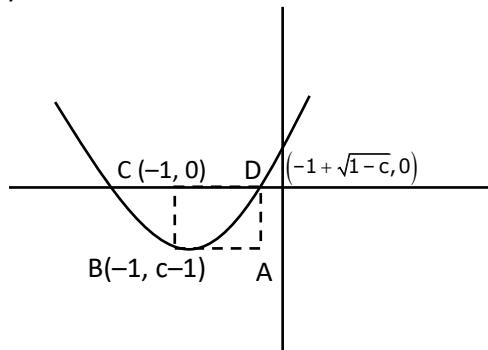
MATRIX

- 10.** Let the curve $y=y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve $y=y(x)$ and x -axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to _____

- 10.** माना अवकल समीकरण $\frac{dy}{dx} = 2(x+1)$ का हल वक्र $y=y(x)$ है। यदि वक्र $y=y(x)$ तथा x -अक्ष से धिरे क्षेत्र के क्षेत्रफल का संख्यात्मक मान $\frac{4\sqrt{8}}{3}$ है तो $y(1)$ का मान बराबर है _____।

Ans. (2)

Sol. $y = x^2 + 2x + c$



$$\text{Area of rectangle (ABCD)} = |(c-1)(\sqrt{1-c})|$$

$$\text{Area of parabola and x-axis} = 2 \left(\frac{2}{3} ((1-c)^{3/2}) \right) = \frac{4\sqrt{8}}{3}$$

$$1 - c = 2 \Rightarrow c = -1$$

$$\text{Equation of } f(x) = x^2 + 2x - 1$$

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$$f(1) = 1 + 2 - 1 = 2$$

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