



**JEE
MAIN
MARCH
2021**

**16thMarch 2021 | Shift - 2
MATHEMATICS**

JEE | NEET | Foundation

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SELECTIONS SINCE 2007

SECTION -A

Complex Number

1. The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\log_e 2\right) \geq \log_{\sqrt{2}}|5\sqrt{7} + 9i|, i = \sqrt{-1}$ is equal to :

(1) 2

(2) 3

(3) 8

(4) $\sqrt{5}$

1. $|z|$, जहाँ z एक समिश्र संख्या है, का न्यूनतम मान, जो असमिका $\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\log_e 2\right) \geq \log_{\sqrt{2}}|5\sqrt{7} + 9i|, i = \sqrt{-1}$ को सन्तुष्ट करता है, है :

(1) 2

(2) 3

(3) 8

(4) $\sqrt{5}$

Ans. (2)

$$\text{Sol. } 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3 \Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$$

$$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$$

$$\Rightarrow |z|^2 - |z| - 6 \geq 0$$

$$(|z| - 3)(|z| + 2) \geq 0$$

$$|z|_{\min} = 3$$

Method of differentiation

2. Let $f : S \rightarrow S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g : S \rightarrow R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(1) $\frac{197}{144}$

(2) $\frac{187}{144}$

(3) $\frac{205}{144}$

(4) 1

2. माना $f : S \rightarrow S$ जहाँ $S = (0, \infty)$ है, दो बार अवकलनीय फलन है जिसके लिए $f(x+1) = xf(x)$ है। यदि $g : S \rightarrow R$ $g(x) = \log_e f(x)$ द्वारा परिभाषित है, तो $|g''(5) - g''(1)|$ का मान बराबर है :

(1) $\frac{197}{144}$

(2) $\frac{187}{144}$

(3) $\frac{205}{144}$

(4) 1

Ans. (3)

Sol. $f(x+1) = xf(x)$

$$g(x+1) = \log_e(f(x+1))$$

$$g(x+1) = \log_e x + \log f(x)$$

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$$g(x+1) - g(x) = \log_e x$$

$$g''(x+1) - g''(x) = -\frac{1}{x^2}$$

$$g''(2) - g''(1) = -1$$

$$g''(3) - g''(2) = -\frac{1}{4}$$

$$g''(4) - g''(3) = -\frac{1}{9}$$

$$g''(5) - g''(4) = -\frac{1}{16}$$

$$g''(5) - g''(1) = -\left[1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right]$$

$$|g''(5) - g''(1)| = \left[\frac{144 + 36 + 16 + 9}{16 \times 9}\right] = \left[\frac{205}{16 \times 9}\right]$$

Differential equation

3. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)y = \sin x, 0 \leq x \leq \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

- (1) $\log_e 2$ (2) $\frac{1}{2} \log_e 2$ (3) $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$ (4) $\frac{1}{4} \log_e 2$

3. यदि अवकल समीकरण $\frac{dy}{dx} + (\tan x)y = \sin x, 0 \leq x \leq \frac{\pi}{3}$ का हल $y = y(x)$ है जबकि $y(0) = 0$ है, तो $y\left(\frac{\pi}{4}\right)$ बराबर है

- (1) $\log_e 2$ (2) $\frac{1}{2} \log_e 2$ (3) $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$ (4) $\frac{1}{4} \log_e 2$

Ans. (3)

Sol. I.f. = $e^{\int \tan x dx}$
 $= e^{\ln[\sec x]}$
 $= \sec x$

Solution of the equation

$$y(\sec x) = \int (\sin x)(\sec x) dx$$

$$\Rightarrow \frac{y}{\cos x} = \ln(\sec x) + c$$

$$\text{Put } x = 0, c = 0$$

$$\therefore y = \cos x \ln(\sec x)$$

$$\text{put } x = \pi/4$$

$$y = \frac{1}{\sqrt{2}} \ln \sqrt{2} = \frac{1}{2\sqrt{2}} \ln 2$$

$$y = \frac{\ln 2}{2\sqrt{2}}$$

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3-D

4. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$, $\ell \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{6}}$

4. यदि रेखा $L_1 : \frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$, $\ell \neq 0$ पर, बिन्दु (4, 3, 8) से लम्ब का पाद (3, 5, 7) है, तो रेखा L_1 तथा रेखा $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ के बीच की न्यूनतम दूरी बराबर है:

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{6}}$

Ans. (4)

Sol. (3, 5, 7) lie on given line L_1

$$\frac{3-a}{\ell} = \frac{3}{3} = \frac{7-b}{4}$$

$$\frac{7-b}{4} = 1 \Rightarrow b = 3$$

$$\frac{3-a}{\ell} = 1 \Rightarrow 3-a = \ell$$

A (4, 3, 8)

B (3, 5, 7)

DR'S of AB = (1, -2, 1)

AB \perp line L_1

$$(1)(\ell) + (-2)(3) + 4(1) = 0$$

$$\Rightarrow \ell = 2$$

$$a = 1$$

$$a = 1, b = 3, \ell = 2$$

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

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$$S.D. = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}} = \frac{1}{\sqrt{6}}$$

3-D

5. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of the expression

$$3 + \frac{x - 11}{(y - 19)^2 (z - 12)^2} + \frac{y - 19}{(x - 11)^2 (z - 12)^2} + \frac{z - 12}{(z - 11)^2 (y - 19)^2} - \frac{x + y + z}{14(x - 11)(y - 19)(z - 12)}$$

to :

- (1) 3 (2) 0 (3) 39 (4) -45

5. माना बिन्दुओं $(42, 0, 0)$, $(0, 42, 0)$ तथा $(0, 0, 42)$ से होकर जाने वाले समतल P पर (x, y, z) एक स्वेच्छ बिन्दु है, तो व्यंजक

$$3 + \frac{x - 11}{(y - 19)^2 (z - 12)^2} + \frac{y - 19}{(x - 11)^2 (z - 12)^2} + \frac{z - 12}{(z - 11)^2 (y - 19)^2} - \frac{x + y + z}{14(x - 11)(y - 19)(z - 12)}$$

- (1) 3 (2) 0 (3) 39 (4) -45

Ans. (1)

Plane passing through $(42, 0, 0)$, $(0, 42, 0)$, $(0, 0, 42)$ form intercept from, equation of plane is

$$x + y + z = 42$$

$$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$$

$$\text{Let } a = x - 11, b = y - 19, c = z - 12$$

$$a + b + c = 0$$

Now, given expression is

$$3 + \frac{a}{b^2 c^2} + \frac{b}{a^2 c^2} + \frac{c}{a^2 b^2} - \frac{42}{14abc}$$

$$3 + \frac{a^3 + b^3 + c^3 - 3abc}{a^2 b^2 c^2}$$

$$\text{If } a + b = c = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow 3 + \frac{a^3 + b^3 + c^3 - 3abc}{a^2 b^2 c^2} = 3$$

Other Method

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equation of plane $x + y + z = 42$

Let pt. on plane $x = 10, y = 21, z = 11$

$$3 + \frac{(-1)}{(4)(1)} + \frac{(2)}{(1)(1)} + \frac{(-1)}{(1)(4)} - \frac{42}{14(-1)(2)(-1)}$$

$$3 - \frac{1}{4} + 2 - \frac{1}{4} - \frac{3}{2} = 3$$

Definite Integration

6. Consider the integral

$$I = \int_0^{10} \frac{[x] e^{[x]}}{e^{x-1}} dx$$

Where $[x]$ denotes the greatest integer less than or equal to x . Then the value of I is equal to :

- (1) $45(e - 1)$ (2) $45(e + 1)$ (3) $9(e - 1)$ (4) $9(e + 1)$

6. समाकलन

$$I = \int_0^{10} \frac{[x] e^{[x]}}{e^{x-1}} dx$$

का विचार कीजिए, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है तो I का मान बराबर है :

- (1) $45(e - 1)$ (2) $45(e + 1)$ (3) $9(e - 1)$ (4) $9(e + 1)$

Ans. (1)

Sol. $I = \int_0^{10} [x] \cdot e^{[x]+1-x} dx$

$$= \int_1^2 e^{2-x} dx + \int_2^3 2 \cdot e^{3-x} dx + \int_3^4 3 \cdot e^{4-x} dx + \dots + \int_9^{10} 9e^{10-x} dx$$

$$= -\{(1-e) + 2(1-e) + 3(1-e) + \dots + 9(1-e)\}$$

$$= 45(e-1)$$

Straight Line

7. Let A (-1, 1), B (3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of $\triangle ABC$ and $\triangle PQC$ respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

- (1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

7. माना तीन बिन्दु A (-1, 1), B (3, 4) तथा C(2, 0) दिये गये हैं। एक रेखा $y = mx$, $m > 0$ रेखाओं AC तथा BC को क्रमशः बिन्दुओं P तथा Q पर काटती है। मान $\triangle ABC$ तथा $\triangle PQC$ के क्षेत्रफल क्रमशः A_1 तथा A_2 हैं जिनके लिए $A_1 = 3A_2$ है, तो m का मान बराबर है :

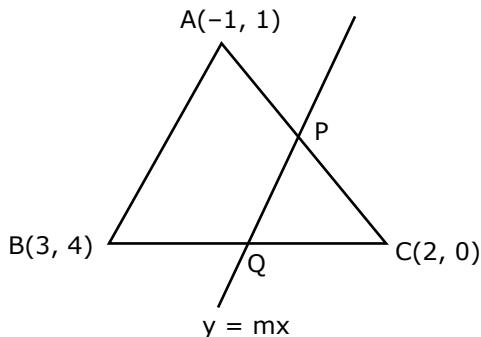
- (1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

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Ans. (2)

Sol.



$$A_1 = \Delta ABC = \frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix}$$

$$A_1 = \frac{13}{2}$$

$$\text{Equation of line } AC \text{ is } y - 1 = -\frac{1}{3}(x + 1)$$

$$\text{solve it with line } y = mx, \text{ we get } P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$$

$$\text{Equation of line } BC \text{ is } y - 0 = 4(x - 2)$$

$$\text{Solve it with line } y = mx, \text{ we get } Q\left(\frac{-8}{m-4}, \frac{-8m}{m-4}\right)$$

$$A_2 = \text{Area of } \triangle PQC = \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{vmatrix} = \frac{A_1}{3} = \frac{13}{6}$$

$$= \frac{1}{2} \left(2 \left(\frac{2m}{3m+1} + \frac{8m}{m-4} \right) - 1 \left(\frac{-16m}{(3m+1)(m-4)} + \frac{16m}{(3m+1)(m-4)} \right) \right)$$

$$= \pm \frac{13}{6}$$

$$\frac{26m^2}{3m^2 - 11m - 4} = \pm \frac{13}{6}$$

$$\Rightarrow 12m^2 = \pm (3m^2 - 11m - 4)$$

taking +ve sign

$$9m^2 + 11m + 4 = 0 \text{ (Rejected as m is imaginary)}$$

taking -ve sign

$$15m^2 - 11m - 4 = 0$$

$$m = 1, -\frac{4}{15}$$

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Monotonicity

8. Let f be a real valued function, defined on $\mathbb{R} - \{-1, 1\}$ and given by

$$f(x) = 3 \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function $f(x)$ is increasing ?

- | | |
|--|--|
| (1) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty \right) - \{1\} \right)$ | (2) $\left(-1, \frac{1}{2} \right]$ |
| (3) $(-\infty, \infty) - \{-1, 1\}$ | (4) $\left(-\infty, \frac{1}{2} \right] - \{-1\}$ |

8. माना $\mathbb{R} - \{-1, 1\}$ पर परिभाशित एक वास्तविक मान फलन f , $f(x) = 3 \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}$

द्वारा दिया गया है। तो फलन $f(x)$ निम्न में से किस अंतराल में वर्धमान है ?

- | | |
|--|--|
| (1) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty \right) - \{1\} \right)$ | (2) $\left(-1, \frac{1}{2} \right]$ |
| (3) $(-\infty, \infty) - \{-1, 1\}$ | (4) $\left(-\infty, \frac{1}{2} \right] - \{-1\}$ |

Ans. (1)

Sol. $f'(x) = \left(\frac{x+1}{x-1} \times \frac{x+1-(x-1)}{(x+1)^2} \right) 3 + \frac{2}{(x-1)^2} = \frac{6}{(x-1)(x+1)} + \frac{2}{(x-1)^2}$
 $= \frac{2}{(x-1)} \left(\frac{3}{x+1} + \frac{1}{x-1} \right) = \frac{4(2x-1)}{(x+1)(x-1)^2}$

+	-	+
-1	$\frac{1}{2}$	

$$x \in (-\infty, -1) \cup \left[\frac{1}{2}, \infty \right) - \{1\}$$

Circle

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, ($a < 0$) be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

- (1) $\sqrt{10}$ (2) $\sqrt{6}$ (3) $\sqrt{11}$ (4) $\sqrt{7}$

9. माना वृत्त $x^2 + y^2 + ax + 2ay + c = 0$, ($a < 0$) द्वारा x-अक्ष तथा y-अक्ष पर बनाये गये अंतःखंडों की लम्बाईयाँ क्रमशः $2\sqrt{2}$ तथा $2\sqrt{5}$ हैं। तो इस वृत्त की एक स्पर्श रेखा, जो रेखा, $x + 2y = 0$ के लम्बवत् है, की मूलबिंदु से न्यूनतम दूरी बराबर है :

- (1) $\sqrt{10}$ (2) $\sqrt{6}$ (3) $\sqrt{11}$ (4) $\sqrt{7}$

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Ans. (2)

$$\text{Sol. } 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$$\sqrt{a^2 - 4c} = 2\sqrt{2}$$

$$a^2 - 4c = 8 \quad \dots (1)$$

$$2\sqrt{a^2 - c} = 2\sqrt{5}$$

$$a^2 - c = 5 \quad \dots (2)$$

$$(2) - (1)$$

$$3c = -3a \Rightarrow c = -1$$

$$a^2 = 4 \Rightarrow a = -2$$

$$x^2 + y^2 - 2x - 4y - 1 = 0$$

$$\text{Equation of tangent } 2x - y + \lambda = 0$$

$$\therefore p = r$$

$$\left| \frac{2 - 2 + \lambda}{\sqrt{5}} \right| = \sqrt{6}$$

$$\Rightarrow \lambda = \pm \sqrt{30}$$

$$\therefore \text{tangent } 2x - y \pm \sqrt{30} = 0$$

$$\text{Distance from origin} = \frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$$

Probability

- 10.** Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

- (1) $\frac{4}{9}$ (2) $\frac{9}{56}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

- 10.** माना A, अंकों 0, 1, 2, 3, 4, 5, 6 द्वारा बिना पुनरावृत्ति के बनाई गई 6-अंकों की संख्या के 3 से विभाजित होने की घटना को दर्शाता है। तो घटना A की प्रायिकता बराबर है :

- (1) $\frac{4}{9}$ (2) $\frac{9}{56}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

Ans. (1)

Sol. Total case = 6|6

$$\begin{aligned} \text{Fav. case} &= (0, 1, 2, 3, 4, 5) + (0, 1, 2, 4, 5, 6) + (1, 2, 3, 4, 5, 6) \\ &= 5|5 + 5|5 + |6 \end{aligned}$$

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= 1920

$$\text{Probability} = \frac{1920}{6|6} = \frac{4}{9}$$

Continuity

- 11.** Let $\alpha \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} \frac{\cos^{-1}(1 - \{x\}^2)\sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3} & x \neq 0 \\ \alpha & x = 0 \end{cases}$ is

Continuous at $x = 0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x .

Then :

- (1) $\alpha = \frac{\pi}{4}$ (2) No such α exists (3) $\alpha = 0$ (4) $\alpha = \frac{\pi}{\sqrt{2}}$

- 11.** माना $\alpha \in \mathbb{R}$ इस प्रकार है कि फलन $f(x) = \begin{cases} \frac{\cos^{-1}(1 - \{x\}^2)\sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3} & x \neq 0 \\ \alpha & x = 0 \end{cases}$

$x = 0$ पर संतत है, जहाँ $\{x\} = x - [x]$, $[x]$ महत्तम पूर्णांक x है। तो

- (1) $\alpha = \frac{\pi}{4}$ (2) इस प्रकार के α का अस्तित्व नहीं है (3) $\alpha = 0$ (4) $\alpha = \frac{\pi}{\sqrt{2}}$

Ans. (2)

$$\text{Sol. } \text{RHL} = \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1 - x^2)\sin^{-1}(1 - x)}{x(1 - x^2)} = \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1 - x^2)}{x}$$

$$= \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{-1}{\sqrt{1 - (1 - x^2)^2}} (-2x) \quad (\text{L' Hospital Rule})$$

$$= \pi \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2x^2 - x^4}} = \pi \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2 - x^2}} = \frac{\pi}{\sqrt{2}}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} \frac{\cos^{-1}(1 - (1 + x)^2)\sin^{-1}(-x)}{(1 + x) - (1 + x)^3} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{(1 + x)[(1 + x)^2 - 1]} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{x^2 + 2x}$$

$$= \frac{\pi}{2} \left(\frac{1}{2}\right) = \frac{\pi}{4}$$

As LHL \neq RHL so $f(x)$ is not continuous at $x = 0$

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DETERMINANT

12. The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in \mathbb{R}$ is :
- (1) $\sqrt{7}$ (2) $\sqrt{5}$ (3) 5 (4) $\frac{3}{4}$

12. $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in \mathbb{R}$ का अधिकतम मान है :
- (1) $\sqrt{7}$ (2) $\sqrt{5}$ (3) 5 (4) $\frac{3}{4}$

Ans. (2)

Sol. $C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$= (-1)[2\sin 2x - \cos 2x] = \cos 2x - 2\sin 2x$$

$$\text{maximum value} = \sqrt{5}$$

Permutation Combination

13. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

- (1) 1890 (2) 795 (3) 717 (4) 1173

13. एक चतुर्भुज ABCD, जिसके रेखा खंडों AB, CD, BC, DA के अन्दर क्रमशः 5, 7, 6, 9 बिन्दु हैं, का विचार कीजिए। माना α उन त्रिभुजों की संख्या है, जिनके भीर्श भिन्न भुजाओं पर ये बिन्दु हैं तथा β उन चतुर्भुजों की संख्या है, जिनके भीर्श भिन्न भुजाओं पर ये बिन्दु हैं। तो $(\beta - \alpha)$ बराबर है :

- (1) 1890 (2) 795 (3) 717 (4) 1173

Ans. (3)

Sol. $\alpha = {}^6C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^7C_1 = 378 + 315 + 270 + 210 = 1173$

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$$\beta = {}^5C_1 {}^6C_1 {}^7C_1 {}^9C_1 = 1890$$

$$\Rightarrow \beta - \alpha = 1890 - 1173 = 717$$

Parabola, Ellipse & Hyperbola

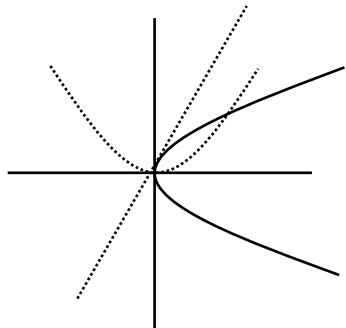
- 14.** Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2, 1) is :

(1) $2x + y = 5$ (2) $x + 2y = 4$ (3) $x + 3y = 5$ (4) $x - y = 1$

- 14.** माना परवलय $y^2 = 4x$ पर एक बिन्दु का रेखा $y = x$ के सापेक्ष दर्पण प्रतिबिम्ब का बिन्दुपथ C है। तो P(2, 1) पर C की स्पर्श रेखा का समीकरण है :

(1) $2x + y = 5$ (2) $x + 2y = 4$ (3) $x + 3y = 5$ (4) $x - y = 1$

Ans. (4)



Sol. Image of $y^2 = 4x$ w.r.t. $y = x$ is $x^2 = 4y$

tangent from (2, 1)

$$xx_1 = 2(y + y_1)$$

$$2x = 2(y + 1)$$

$$x = y + 1$$

Inverse Trigonometric Functions

- 15.** Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$ is equal to :

(1) 1 (2) 2 (3) 3 (4) 0

- 15.** यह दिया गया है कि प्रतिलोम त्रिकोणमितीय फलन केवल मुख्य मान लेते हैं। तो $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$ को सन्तुष्ट करने वाले x के वास्तविक मानों की संख्या है :

(1) 1 (2) 2 (3) 3 (4) 0

Ans. (3)

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Sol. Taking sine both sides

$$\begin{aligned} \frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} &= x \\ \Rightarrow 3x\sqrt{25 - 16x^2} &= 25x - 4x\sqrt{25 - 9x^2} \\ \Rightarrow x = 0 \text{ or } 3\sqrt{25 - 16x^2} &= 25 - 4\sqrt{25 - 9x^2} \\ \Rightarrow 9(25 - 16x^2) &= 625 - 200\sqrt{25 - 9x^2} + 16(25 - 9x^2) \\ \Rightarrow 200\sqrt{25 - 9x^2} &= 800 \\ \Rightarrow \sqrt{25 - 9x^2} &= 4 \\ \Rightarrow x^2 &= 1 \\ \Rightarrow x &\pm 1 \\ \therefore \text{Total number of solution} &= 3 \end{aligned}$$

Differential equation

- 16.** Let C_1 be the curve obtained by solution of differential equation $2xy \frac{dy}{dx} = y^2 - x^2, x > 0$ Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$, If both the curves pass through $(1, 1)$ then the area enclosed by the curves C_1 and C_2 is equal to :

- (1) $\frac{\pi}{2} - 1$ (2) $\frac{\pi}{4} + 1$ (3) $\pi - 1$ (4) $\pi + 1$

- 16.** माना अवकल समीकरण $2xy \frac{dy}{dx} = y^2 - x^2, x > 0$ का हल वक्र C_1 है तथा $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$ का हल वक्र C_2 है। यदि दोनों वक्र $(1, 1)$ से होकर जाते हैं, तो वक्रों C_1 तथा C_2 द्वारा परिबद्ध क्षेत्र का क्षेत्रफल बराबर है :

- (1) $\frac{\pi}{2} - 1$ (2) $\frac{\pi}{4} + 1$ (3) $\pi - 1$ (4) $\pi + 1$

Ans. (1)

Sol.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put $y = vx$

$$\begin{aligned} v + x \frac{dv}{dx} &= \frac{v^2 x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v} \\ x \frac{dv}{dx} &= \frac{v^2 - 1 - 2v^2}{2v} = -\frac{(v^2 + 1)}{2v} \end{aligned}$$

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$$\Rightarrow \frac{2v}{v^2 + 1} dv = -\frac{dx}{x}$$

$$\ell n(v^2 + 1) = -\ell nx + \ell nc \Rightarrow v^2 + 1 = \frac{c}{x}$$

$$\Rightarrow \frac{y^2}{x^2} + 1 = \frac{c}{x} \Rightarrow x^2 + y^2 = cx$$

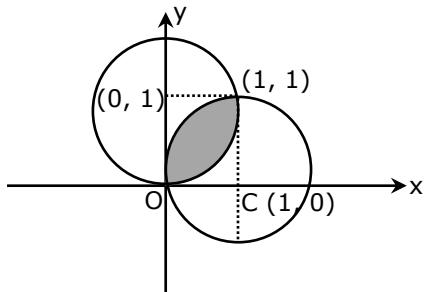
If pass through (1,1)

$$\therefore x^2 + y^2 - 2x = 0$$

Similarly for second differential equation $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Equation of curve is $x^2 + y^2 - 2y = 0$

Now required area is



$$= \left(\frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1 \right) \times 2$$

$$= \left(\frac{\pi}{2} - 1 \right) \text{ sq. units}$$

VECTOR

17. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$ and

$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$, $\alpha \in \mathbb{R}$, then the value of $\alpha + |\vec{r}|^2$ is equal to :

- (1) 11 (2) 15 (3) 9 (4) 13

17. माना $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ तथा $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ हैं। यदि $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$ तथा

$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$, $\alpha \in \mathbb{R}$ है, तो $\alpha + |\vec{r}|^2$ का मान बराबर है :

- (1) 11 (2) 15 (3) 9 (4) 13

Ans. (2)

Sol. $\vec{r} \times \vec{a} = -\vec{r} \times \vec{b}$

$$\vec{r} \times (\vec{a} + \vec{b}) = 0 \quad (\vec{a} + \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k})$$

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$$\vec{r} \parallel (\vec{a} + \vec{b})$$

$$\vec{r} = \lambda(\vec{a} + \vec{b})$$

$$\therefore \vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$$

$$\lambda[3\hat{i} - \hat{j} + 2\hat{k}] \cdot [2\hat{i} + 5\hat{j} - \alpha\hat{k}] = -1$$

$$\Rightarrow \lambda(6 - 5 - 2\alpha) = -1$$

$$\lambda(1 - 2\alpha) = -1 \quad \dots(1)$$

$$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\lambda(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\Rightarrow \lambda[3\alpha - 2 + 2] = 3 \Rightarrow \lambda\alpha = 1 \quad \dots(2)$$

(1) & (2)

$$\lambda\left[1 - \frac{2}{\lambda}\right] = -1$$

$$\lambda - 2 = -1 \Rightarrow \lambda = 1 \quad \alpha = 1$$

$$\vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\alpha + |\vec{r}|^2 \Rightarrow 1 + 14 = 15$$

Definite Integration

- 18.** Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x) dx = 1$

and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b+c)$ is equal to :

- (1) 7 (2) 11 (3) 15 (4) 9

- 18.** माना वास्तविक गुणांकों का एक द्विघातीय बहुपद $P(x) = x^2 + bx + c$ इस प्रकार है कि $\int_0^1 P(x) dx = 1$ है तथा $P(x)$ को

$(x - 2)$ से विभाजित करने पर शेषफल 5 आता है। तो $9(b+c)$ का मान बराबर है :

- (1) 7 (2) 11 (3) 15 (4) 9

Ans. (1)

Sol. $(x - 2)Q(x) + 5 = x^2 + bx + c$

Put $x = 2$

$$5 = 2b+c+4 \quad \dots(1)$$

$$\int_0^1 (x^2 + bx + c) dx = 1$$

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$$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$$

$$\frac{b}{2} + c = \frac{2}{3} \dots(2)$$

Solve (1) & (2)

$$b = \frac{2}{9}$$

$$c = \frac{5}{9}$$

$$9(b+c) = 7$$

Parabola, Ellipse & Hyperbola

- 19.** If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to :

(1) 5 (2) 6 (3) 12 (4) 10

- 19.** यदि दीर्घवृत्त $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ तथा वृत्त $x^2 + y^2 = 4b$, $b > 4$ के प्रतिच्छेदन बिन्दु वक्र $y^2 = 3x^2$ पर स्थित हैं, तो b बराबर है :

(1) 5 (2) 6 (3) 12 (4) 10

Ans. (3)

Sol. $\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \dots(1)$

$$x^2 + y^2 = 4b \dots(2)$$

$$y^2 = 3x^2 \dots(3)$$

From eq (2) and (3) $x^2 = b$ and $y^2 = 3b$

From equation (1) $\frac{b}{16} + \frac{3b}{b^2} = 1$

$$\Rightarrow b^2 + 48 = 16b$$

$$\Rightarrow b = 12$$

Parabola, Ellipse & Hyperbola

- 20.** Let A = {2, 3, 4, 5, ..., 30} and '□' be an equivalence relation on A×A, defined by (a, b) □ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 7 (2) 5 (3) 6 (4) 8

- 20.** माना A = {2, 3, 4, 5, ..., 30} है तथा A×A पर, (a, b) □ (c, d), यदि और केवल यदि ad = bc है, द्वारा परिभाषित एक तुल्यता संबंध '≈' है। तो क्रमित युग्मों की संख्या, जो क्रमित युग्म (4, 3) के साथ इस तुल्यता संबंध को सन्तुष्ट करते हैं, है

(1) 7 (2) 5 (3) 6 (4) 8

Ans. (1)

Sol. ad = bc

$$(a, b) R (4, 3) \Rightarrow 3a = 4b$$

$$a = \frac{4}{3} b$$

b must be multiple of 3

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$$b = \{3, 6, 9, \dots, 30\}$$

$$(a, b) = (4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)$$

\Rightarrow 7 ordered pair

SECTION -B

VECTOR

- Let \vec{c} be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$ then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to
- माना सदिशों $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ तथा $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ के लम्बवत एक सदिश \vec{c} है। यदि $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$ है, तो $\vec{c} \cdot (\vec{a} \times \vec{b})$ का मान बराबर है

Ans. (28)

Sol. $\vec{c} \cdot (\vec{a} \times \vec{b}) = [\vec{c} \vec{a} \vec{b}]$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (3, -2, 1)$$

$$\vec{c} \perp \vec{a}, \vec{c} \perp \vec{b} \Rightarrow \vec{c} \parallel \vec{a} \times \vec{b}$$

$$\vec{c} = \lambda (\vec{a} \times \vec{b})$$

$$\Rightarrow \vec{c} = \lambda (3\hat{i} - 2\hat{j} + \hat{k})$$

$$\vec{c}(\hat{i} + \hat{j} + 3\hat{k}) = 8$$

$$\Rightarrow 3\lambda - 2\lambda + 3\lambda = 8$$

$$\Rightarrow 4\lambda = 8 \Rightarrow \lambda = 2$$

$$\vec{c} = 6\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{c} \cdot (\vec{a} \times \vec{b}) = [\vec{c} \vec{a} \vec{b}] = \begin{vmatrix} 6 & -4 & 2 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow 18 + 8 + 2 = 28$$

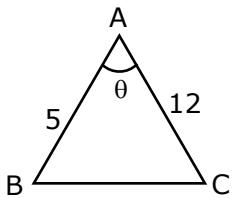
Solution of Triangle

- In $\triangle ABC$, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of $\triangle ABC$ is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of $\triangle ABC$, then the value of $2R + r$ (in cm) is equal to
- एक त्रिभुज ABC में, भुजाओं AC तथा AB की लम्बाई क्रमशः 12 cm तथा 5 cm है। यदि $\triangle ABC$ का क्षेत्रफल 30 cm^2 है तथा $\triangle ABC$ के परिवृत्त और अंतवृत्त की त्रिज्यायें क्रमशः R और r हैं, तो $2R + r$ का मान (cm में) बराबर है

Ans. (15)

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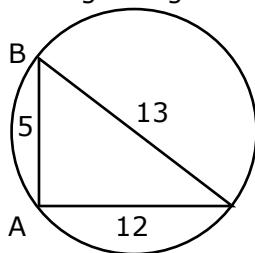


Sol.

$$\text{Area} = \frac{1}{2}(5)(12)\sin\theta = 30$$

$$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$$

Δ is right angle Δ



$$r = (s-a) \tan \frac{A}{2}$$

$$r = (s-a)$$

$$r = (s-a) \quad (a = 2R)$$

$$2R + r = s$$

$$2R + r = \frac{30}{2} = 15$$

Central tendency & differentiation

3. Consider the statistics of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal

to

3. नीचे दो गई प्रेक्षणों के दो समूहों की सांख्यिकी का विचार कीजिए :

	आकार	माध्य	प्रसरण
प्रेक्षण I	10	2	2
प्रेक्षण II	n	3	1

यदि इन दोनों प्रेक्षणों को मिलाकर बने समूह का प्रसरण $\frac{17}{9}$ है, तो n का मान बराबर है

Ans. (5)

Sol. For group-1 : $\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$

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$$\frac{\sum x_i}{10} - (2)^2 = 2 \Rightarrow \sum x_i^2 = 60$$

$$\text{For group-2 : } \frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$$

$$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$$

Now, combined variance

$$\sigma^2 = \frac{\sum (x_i^2 + y_i^2)}{10+n} - \left(\frac{\sum (x_i + y_i)}{10+n} \right)^2$$

$$\Rightarrow \frac{17}{9} = \frac{60+10n}{10+n} - \frac{(20+3n)^2}{(10+n)^2}$$

$$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0 \Rightarrow n = 5$$

Central tendency & differentiation

4. Let $S_n(x) = \log_{\frac{1}{a^2}} x + \log_{\frac{1}{a^3}} x + \log_{\frac{1}{a^6}} x + \log_{\frac{1}{a^{11}}} x + \log_{\frac{1}{a^{18}}} x + \dots$ up to n-terms,

Where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, the value of a is equal to

4. माना $S_n(x) = \log_{\frac{1}{a^2}} x + \log_{\frac{1}{a^3}} x + \log_{\frac{1}{a^6}} x + \log_{\frac{1}{a^{11}}} x + \log_{\frac{1}{a^{18}}} x + \dots$ n पदों तक, जहाँ $a > 1$ है। यदि

$S_{24}(x) = 1093$ तथा $S_{12}(2x) = 265$ हैं, तो a का मान बराबर है।

Ans. (16)

$$S_n(x) = \log_a x^2 + \log_a x^3 + \log_a x^6 + \log_a x^{11}$$

$$S_n(x) = 2 \log_a x + 3 \log_a x + 6 \log_a x + 11 \log_a x + \dots$$

$$S_n(x) = \log_a x (2 + 3 + 6 + 11 + \dots)$$

$$S_r = 2 + 3 + 6 + 11$$

$$\text{General term } T_r = r^2 - 2r + 3$$

$$S_n(x) = \sum_{r=1}^n \log_a x (r^2 - 2r + 3)$$

$$S_{24}(x) = \sum_{r=1}^{24} \log_a x (r^2 - 2r + 3)$$

$$S_{24}(x) = \log_a \sum_{r=1}^{24} (r^2 - 2r + 3)$$

$$1093 = 4372 \log_a x$$

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$$\log_a x = \frac{1}{4}$$

$$x = a^{1/4}$$

$$S_{12}(2x) = \log_a(2x) \sum_{r=1}^{12} (r^2 - 2r + 3)$$

$$265 = 530 \log_a(2x)$$

$$\log_a(2x) = \frac{1}{2}$$

$$2x = a^{1/2}$$

After solving (i) and (ii), we get

$$a^{1/4} = 2$$

$$a = 16$$

Binomial Theorem

5. Let n be a positive integer. Let $A = \sum_{k=0}^n (-1)^k nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$.

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to

5. माना n एक धनात्मक पूर्णांक है तथा $A = \sum_{k=0}^n (-1)^k nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$

है। यदि $63A = 1 - \frac{1}{2^{30}}$ है तो, n बराबर है

Ans. (6)

$$\text{Sol. } A = \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n + \left(\frac{1}{32}\right)^n$$

$$= \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \frac{1}{2^{5n}}$$

$$= \frac{1}{2^n} \left[\frac{1 - \left(\frac{1}{2^n}\right)^5}{1 - \frac{1}{2^n}} \right]$$

$$A = \frac{2^{5n} - 1}{2^{5n}(2^n - 1)}$$

$$63A = \frac{63(2^{5n} - 1)}{2^{5n}(2^n - 1)}$$

$$\frac{63}{2^n - 1} \left(1 - \frac{1}{2^{5n}}\right) = 63A = \left(1 - \frac{1}{2^{30}}\right)$$

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$$= \frac{63}{2^n - 1} \left(1 - \frac{1}{2^{5n}} \right) = \left(1 - \frac{1}{2^{30}} \right)$$

$$n = 6$$

Continuity

6. Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined as

$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 1)^2 + b, & x \geq 0 \end{cases}$$

Where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x \in R$, then $a + b$ is equal to

6. माना $f : R \rightarrow R$ तथा $g : R \rightarrow R$

$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases} \text{ तथा } g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 1)^2 + b, & x \geq 0 \end{cases}$$

द्वारा परिभाषित है, जहाँ a, b ऋणेतर वास्तविक संख्यायें हैं। यदि $(gof)(x)$ सभी $x \in R$ के लिए संतत है, तो $a + b$ बराबर है

Ans. (1)

$$\begin{aligned} \text{Sol. } g[f(x)] &= \begin{cases} f(x) + 1 & f(x) < 0 \\ (f(x) - 1)^2 + b & f(x) \geq 0 \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x + a < 0 \& x < 0 \\ |x - 1| + 1 & |x - 1| < 0 \& x \geq 0 \\ (x + a - 1)^2 + b & x + a \geq 0 \& x < 0 \\ (|x - 1| - 1)^2 + b & |x - 1| \geq 0 \& x \geq 0 \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x \in (-\infty, -a) \& x \in (-\infty, 0) \\ |x - 1| + 1 & x \in \phi \\ (x + a - 1)^2 + b & x \in [-a, \infty) \& x \in (-\infty, 0) \\ (|x - 1| - 1)^2 + b & x \in R \& x \in [0, \infty) \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x \in (-\infty, -a) \\ (x + a - 1)^2 + b & x \in [-a, 0) \\ (|x - 1| - 1)^2 + b & x \in [0, \infty) \end{cases} \end{aligned}$$

$g(f(x))$ is continuous

at $x = -a$ & at $x = 0$

$$1 = b + 1 \& (a-1)^2 + b = b$$

$$b = 0 \& a = 1$$

$$\Rightarrow a + b = 1$$

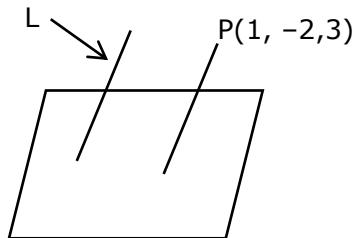
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3-D

7. If the distance of the point $(1, -2, 3)$ from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to
 7. यदि समतल $x + 2y - 3z + 10 = 0$ से बिन्दु $(1, -2, 3)$ की रेखा $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ के समान्तर दूरी $\sqrt{\frac{7}{2}}$ है, तो $|m|$ का मान बराबर है

Ans. (2)



Sol.

$$\frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$$

Pt. Q $(3\lambda + 1, -m\lambda - 2, \lambda + 3)$ lie on plane

$$(3\lambda + 1) + 2(-m\lambda - 2) - 3(\lambda + 3) + 10 = 0$$

$$\Rightarrow 3\lambda - 2m\lambda - 3\lambda + 1 - 4 - 9 + 10 = 0$$

$$\Rightarrow -2m\lambda = 2$$

$$m\lambda = -1 \Rightarrow \lambda = -\frac{1}{m}$$

$$Q\left[-\frac{3}{m} + 1, -1, -\frac{1}{m} + 3\right]$$

$$PQ = \sqrt{\frac{7}{2}}$$

$$\sqrt{\left(-\frac{3}{m}\right)^2 + 1 + \left(-\frac{1}{m}\right)^2} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow \frac{10 + m^2}{m^2} = \frac{7}{2}$$

$$\Rightarrow 20 + 2m^2 = 7m^2$$

$$m^2 = 4 \Rightarrow |m| = 2$$

DETERMINANT

8. Let $\frac{1}{16}, a$ and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P. where $a, b, > 0$. Then $72(a+b)$ is equal to
 8. माना $\frac{1}{16}, a$ तथा b G.P. में हैं तथा $\frac{1}{a}, \frac{1}{b}, 6$ A.P. में हैं, $a, b, > 0$ हैं। तो $72(a+b)$ बराबर है

Ans. (14)

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Sol. $a^2 = \frac{b}{16}$ and $\frac{2}{b} = \frac{1}{a} + 6$

Solving, we get $a = \frac{1}{12}$ or $a = -\frac{1}{4}$ [rejected]

if $a = \frac{1}{12} \Rightarrow b = \frac{1}{9}$

$$\therefore 72(a+b) = 72\left(\frac{1}{12} + \frac{1}{9}\right) = 14$$

MATRIX

9. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where $x = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$ then the value of k is...

9. माना 2×1 के दो आव्यूह $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ तथा $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ हैं जिनके अवयव वास्तविक हैं तथा $A = XB$ है, जहाँ $x = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ और $k \in R$ है। यदि $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ तथा $(k^2 + 1)b_2^2 \neq -2b_1b_2$ है, तो k का मान है...।

Ans. (1)

Sol.

$$XB = A$$

$$\frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 - b_2 = \sqrt{3}a_1 \Rightarrow 3a_1^2 = b_1^2 + b_2^2 - 2b_1b_2$$

$$b_1 + kb_2 = \sqrt{3}a_2 \Rightarrow 3a_2^2 = b_1^2 + k^2b_2^2 + 2kb_1b_2$$

$$3(a_1^2 + a_2^2) = 2b_1^2 + (k^2 + 1)b_2^2 + 2b_1b_2(k - 1)$$

$$a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{(k^2 + 1)}{3}b_2^2 + \frac{2}{3}b_1b_2(k - 1)$$

given

$$a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\frac{2}{3}(b_1^2 + b_2^2) = \frac{2}{3}b_1^2 + \frac{(k^2 + 1)}{3}b_2^2 + \frac{2}{3}b_1b_2(k - 1)$$

$$\frac{2}{3}b_2^2 = \frac{(k^2 + 1)}{3}b_2^2 + \frac{2}{3}b_1b_2(k - 1)$$

compare both sides, we get

$$\frac{(k^2 + 1)}{3} = \frac{2}{3} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1 \quad \dots\dots(1)$$

$$\text{and } \frac{2}{3}(k - 1) = 0 \Rightarrow k = 1 \quad \dots\dots(2)$$

from (1) and (2)

$$k = 1$$

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Indefinite Integration

10. For real number α, β, γ and δ , if

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx \\ = \alpha \log_e\left(\tan^{-1}\left(\frac{x^2 + 1}{x}\right)\right) + \beta \tan^{-1}\left(\frac{\gamma(x^2 - 1)}{x}\right) + \delta \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$$

Where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to

10. वास्तविक संख्याओं α, β, γ तथा δ के लिए, यदि

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx \\ = \alpha \log_e\left(\tan^{-1}\left(\frac{x^2 + 1}{x}\right)\right) + \beta \tan^{-1}\left(\frac{\gamma(x^2 - 1)}{x}\right) + \delta \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$$

है, जहाँ C एक स्वेच्छ अचर है, तो $10(\alpha + \beta\gamma + \delta)$ का मान बराबर है

Ans. (6)

Sol. $\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx + \int \frac{1}{x^4 + 3x^2 + 1} dx$

$$\int \frac{1 - \frac{1}{x^2}}{\left[\left(x + \frac{1}{x}\right)^2 + 1\right]\tan^{-1}\left(x + \frac{1}{x}\right)} dx + \int \frac{dx}{x^4 + 3x^2 + 1}$$

\downarrow \downarrow
 I_1 I_2

$$\tan^{-1}\left(x + \frac{1}{x}\right) = t$$

$$I_1 = \int \frac{dt}{t}$$

$$I_1 = \ln(t) = \ln\left|\tan^{-1}\left(x + \frac{1}{x}\right)\right|$$

Now

$$I_2 = \int \frac{dx}{x^4 + 3x^2 + 1}$$

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$$\begin{aligned}
 &= \frac{1}{2} \int \frac{(x^2 + 1) - (x^2 - 1)}{x^4 + 3x^2 + 1} dx \\
 &= \frac{1}{2} \left[\int \frac{1 + \frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx - \int \frac{\left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx \right] \\
 &\quad \left[\int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx \right] \\
 &= \quad \downarrow \quad \quad \quad \downarrow \\
 &\quad x - \frac{1}{x} = u \quad \quad \quad x + \frac{1}{x} = v \\
 &= \frac{1}{2} \left[\int \frac{du}{u^2 + (\sqrt{5})^2} - \int \frac{dv}{v^2 + 1} \right] \\
 I_2 &= \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{5}} \right) - \frac{1}{2} \tan^{-1} \left(x + \frac{1}{x} \right) \\
 I &= I_1 + I_2 = \ln \left| \tan^{-1} \left(x + \frac{1}{x} \right) \right| + \frac{1}{2\sqrt{5}} \ln \left(\frac{x^2 - 1}{\sqrt{5}x} \right) - \frac{1}{2} \tan^{-1} \left(\frac{x^2 + 1}{x} \right) + C \\
 \alpha &= 1, \beta = \frac{1}{2\sqrt{5}}, \lambda = \frac{1}{\sqrt{5}}, \delta = -\frac{1}{2} \\
 10(\alpha + \beta\lambda + \delta) &= 10 \left[1 + \frac{1}{10} - \frac{1}{2} \right]
 \end{aligned}$$

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